1. Una recently purchased two boxes of ten-inch candles – one box from a discount store, and the other from an expensive boutique. One evening, Una noticed that the inexpensive candles last only three hours each while the expensive candles last five hours each. The next evening, Una hosted a dinner party and lit two candles – one from each box – at 7:30 pm. During dessert, a guest noticed that one candle was twice as long as the other. At what time was this observation made?

2. Rewrite the equation $3x - 5y = 30$ in the form $ax + by = 1$. Are there lines whose equations *cannot* be rewritten in this form?

3. What is the relationship between the lines described by the equations $-20x + 12y = 36$ and $-35x + 21y = 63$? Find a third equation in the form $ax + by = 90$ that fits this pattern.

4. Pat races at 10 miles per hour, while Kim races at 9 miles per hour. When they both ran in the same long distance race last week, Pat finished 8 minutes ahead of Kim. What was the length of the race, in miles? Briefly describe your reasoning.

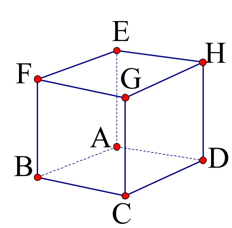
5. (Continuation) Assume that Pat and Kim run at p and k miles per hour, respectively, and that Pat finishes m minutes before Kim. Find the length of the race, in miles.

6. A debt of $450 is to be shared equally among the members of the Outing Club. When five of the members refuse to pay, the other members' shares each go up by $3. How many members does the Outing Club have?

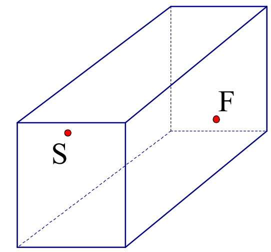
7. Ashley saved a distance equal to 80% of the length of the shortest side of a rectangular field by cutting across the diagonal of the field instead of along two of the sides. Find the ratio of the length of the shortest side of the field to the length of its longest side.

8. A castle is surrounded by a rectangular moat, which is of uniform width 12 feet. A corner is shown in the top view at right. The problem is to get across the moat to the dry land on the other side, without being able to use the drawbridge. All you have to work with are two rectangular planks, whose lengths are 11 feet and 11 feet, 9 inches. Find a way to get across.

9. An ant is positioned at F, one of the eight vertices of a solid cube. It needs to crawl to vertex D as fast as possible. Find one of the shortest routes. How many are there?



10. Given that $ABCDEFGHI$ is a regular polygon, prove that AD and FI have the same length.

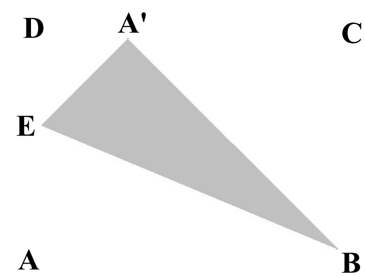


1. A spider lived in a room that measured 30 feet long by 12 feet wide by 12 feet high. One day, the spider spied an incapacitated fly across the room, and of course wanted to crawl to it as quickly as possible. The spider was on an end wall, one foot from the ceiling and six feet from each of the long walls. The fly was stuck one foot from the floor on the opposite wall, also midway between the two long walls. Knowing some geometry, the spider cleverly took the shortest possible route to the fly and ate it for lunch. How far did the spider crawl?

2. Consider the following process for bisecting an angle *ABC*: First mark *M* on *BA* and *P* on *BC* so that *MB* = *PB*, then mark new points *N* on *BA* and *Q* on *BC* so that *NB* = *QB*. Let *E* be the intersection of *MQ* and *NP*. Prove that segment *BE* is the desired angle bisector.

3. Given parallelogram *ABCD*, with diagonals *AC* and *BD* intersecting at *O*, let *POQ* be any line with *P* on *AB* and *Q* on *CD*. Prove that *AP* = *CQ*.

4. Suppose that *ABCD* is a rhombus and that the bisector of angle *BDC* meets side *BC* at *F*. Prove that angle *DFC* is three times the size of angle *FDC*.

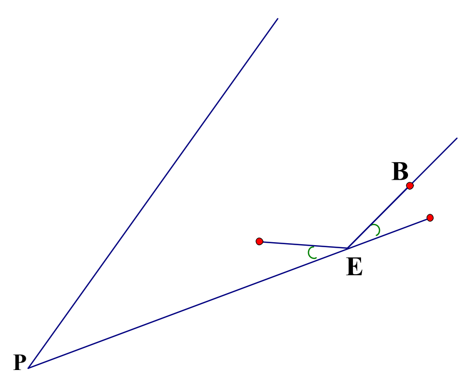


5. In the diagram at right, a rectangular sheet of paper *ABCD* has been creased so that corner *A* is now placed on edge *CD*, at *A*'. Find the size of angle *DEA*', given that the size of angle *ABE* is **(a)** 30°; **(b)** 27°; **(c)** *n*°.

6. Let *ABCD* be a square. Mark midpoints *M*, *N*, *O*, and *P* on *AB*, *BC*, *CD*, and *DA*, respectively. Draw *AN*, *BO*, *CP*, and *DM*. Let *Q* and *R* be the intersections of *AN* with *DM* and *BO*, respectively, and let *S* and *T* be the intersections of *CP* with *BO* and *DM*, respectively. Prove as much as you can about this figure, especially quadrilateral *QRST*.

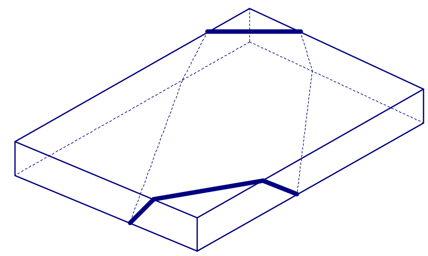
7. (Continuation) Segment *AB* is 10 cm long. How long is *QR*, to the nearest 0.1 cm?

8. The diagram at right shows one corner of a triangular billiards table. A ball leaves point *B* and follows the indicated path, striking the edge of the table at *E*. Thereafter, at each impact, the ball obeys the *law of reflection*, which says that the incoming angle equals the outgoing angle. Given that there is a 34-degree angle at corner *P*, and that the initial impact makes a 25-degree angle at *E*, how many bounces will the ball make before its path leaves this page?



1. The parallel bases of a trapezoid have lengths 12 and 18 cm. Find the lengths of the two segments into which the midline of the trapezoid is divided by a diagonal.

2. The diagonals of a non-isosceles trapezoid divide the midline into three segments whose lengths are 8 cm, 3 cm, and 8 cm. How long are the parallel sides? From this information, is it possible to infer anything about the distance that separates the parallel sides? Explain.



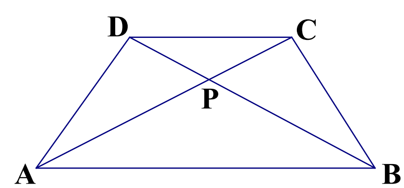
3. What is the smallest amount of ribbon that is needed to wrap around a 2’’ × 10’’ × 20’’ gift box in the way shown in the figure at right? You could experiment with some string and a book.

4. To the nearest tenth of a degree, find the sizes of the acute angles in a 5-12-13 triangle and in a 9-12-15 triangle. This enables you to calculate the sizes of the angles in a 13-14-15 triangle. Show how to do it then invent another example of this sort.

5. In a triangle with vertices (0, 0), (0, 4), (3, 0),
(a) What is the slope of the bisector of the right angle?
(b) What is the equation of the line for the bisector of the larger of the two acute angles?

6. The parallel sides of a trapezoid have lengths 9 cm and 12 cm. Draw *one* diagonal, dividing the trapezoid into two triangles. What is the ratio of their areas? If the other diagonal had been drawn instead, would this have affected your answer?

7. Suppose that *ABCD* is a trapezoid with *AB* parallel to *CD* and diagonals *AC* and *BD* intersecting at *P*. Explain why
(a) triangles *CDA* and *CDB* have the same area;
(b) triangles *BCP* and *DAP* have the same area;
(c) triangles *ABP* and *CDP* are similar;
(d) triangles *BCP* and *DAP* need not be similar.



8. A trapezoid has 11-inch and 25-inch parallel sides, and an area of 216 square inches.
(a) How far apart are the parallel sides?
(b) If one of the non-parallel sides is 13 inches long, how long is the other one? (Note:
There are two answers to this question. It is best to make a separate diagram for each).

1. Let $A = (0, 0)$, $B = (12, 0)$, $C = (8, 6)$, and $D = (2, 6)$. The diagonals *AC* and *BD* of trapezoid *ABCD* intersect at *P*. Explain why triangles *ABP* and *CDP* are similar. What is the ratio of similarity? Which side of triangle *CDP* corresponds to side *AP* in triangle *ABP*? Why is it inaccurate to write $ABP \sim DCP$? Without finding the coordinates of *P*, show how you can find the lengths *AP* and *PC*.

2. (Continuation) Find the ratio of the areas of triangles **(a)** *ADP* and *CDP*; **(b)** *ADP* and *ABP*; **(c)** *CDP* and *ABP*.

3. Draw an accurate version of a regular pentagon. Be prepared to report on the method you used to draw this figure. Measure the length of a diagonal and the length of a side. Then divide the diagonal length by the side length. When you and your classmates compare answers, on which of the preceding numbers should you agree – the lengths or the ratio?

4. (Continuation) Calculate the ratio of the diagonal length to the side length in any regular pentagon. One way to do it is to use trigonometry.

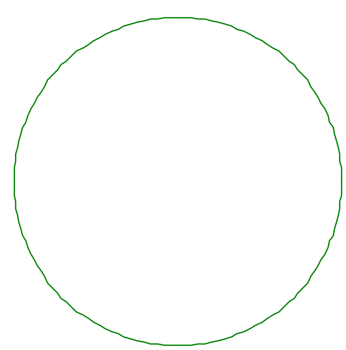
5. (Continuation) Label your pentagon *ABCDE*. Draw its diagonals. They intersect to form a smaller pentagon *A'B'C'D'E'* that lies inside *ABCDE*.
(a) Explain why *A'B'C'D'E'* is regular, and why it is similar to *ABCDE*.
(b) Measure the length *A'B'*, and divide it by *AB*. Then use trigonometry to find an exact value for *A'B'* **:** *AB*, which is called the *ratio of similarity*.
(c) Consider the ways of assigning the labels *A'*, *B'*, *C'*, *D'*, and *E'* to the vertices of the small pentagon. How many ways are there? Is there one that stands out from the rest?

6. (Continuation) It should be possible to *circumscribe* a circle around your pentagon *ABCDE*, meaning that a circle can be drawn that goes through all five of its vertices. Find the center of this circle, and describe your method. Then measure the radius of the circle, and express your answer as a multiple of the length *AB*. Which of these numbers will be more useful to bring to class – the radius or the ratio?

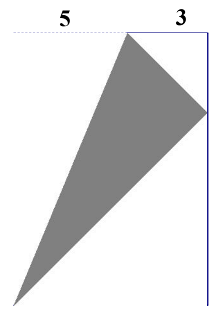
7. Show that the medians of any triangle divide the triangle into six smaller triangles of equal area. Are any of the small triangles necessarily congruent to each other?

8. The area of a trapezoidal cornfield *IOWA* is 18000 sq m. The 100-meter side *IO* is parallel to the 150-meter side *WA*. This field is divided into four sections by diagonal roads *IW* and *OA*. Find the areas of the triangular sections.

9. Explain how to find the center of the circle shown, using only a pencil and a rectangular sheet of paper.



1. Let $A = (0, 0)$, $B = (7, 0)$, and $C = (7, 5)$. Point D is located so that angle ACD is a right angle and the tangent of angle DAC is $\frac{5}{7}$. Find coordinates for D.



2. The figure at right shows a rectangular sheet of paper that has been creased so that one of its corners matches a point on a non-adjacent edge. Given the dimensions marked on the figure, you are to determine the length of the crease.

3. Dana takes a paper cone with a 10-cm diameter and is 12 cm deep, cuts it along a straight line from the rim to the vertex, then flattens the paper out on a table. Find the radius, the arc length, and the central angle of the resulting circular sector.

4. Dana takes a sheet of paper, cuts a 120-degree circular sector from it, then rolls it up and tapes the straight edges together to form a cone. Given that the sector radius is 12 cm, find the height and volume of this paper cone.

5. The diameter and the slant height of a cone are both 24 cm. Find the radius of the largest sphere that can be placed inside the cone. (The sphere is therefore tangent to the base of the cone.)

6. Find the volume of a cone of height 8 centimeters and base radius 6 centimeters. This cone is sliced by a plane that is parallel to the base and 2 centimeters from it. Find the volumes of the two resulting solids. One is a cone, while the other is called a *frustum*.