

Motivational Problems for Double and Half Angle Identities
Precalculus
Ms. Schettino

Double Angle Identities:

1. An isosceles triangle has two sides of length w that make a $2a$ -degree angle. Write down two different formulas for the *area* of this triangle, in terms of w and a . By equating the formulas, discover a relation involving $\sin 2a$, $\sin a$, and $\cos a$.
2. Use the sum identity for cosine to derive the double angle identity for cosine. To do this, let $\cos(2\alpha) = \cos(\alpha + \alpha)$.
3. There are actually three useful forms of the double angle identity for cosine. Recall the Pythagorean identity $\sin^2 \alpha + \cos^2 \alpha = 1$. How could you use these to substitute into the previous problem to obtain 2 more forms of the double angle identity for cosine?

Half Angle Identities:

4. Using the form of the double angle identity for cosine $\cos(2\alpha) = 1 - \sin^2 \alpha$. Substitute $\alpha = \frac{x}{2}$ into this identity. With a little algebraic manipulation, you will get an expression for $\sin(\frac{x}{2})$.
5. Now use the other one, $\cos(2\alpha) = 2 \cos^2 \alpha - 1$ and again substitute $\alpha = \frac{x}{2}$. A little more algebra again, and you will get an expression for $\cos(\frac{x}{2})$.
6. Find an exact values for $\sin(\frac{\pi}{8})$ and $\cos(\frac{\pi}{8})$.