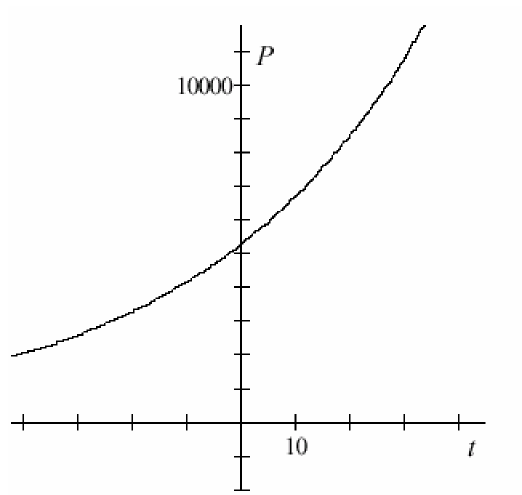
Motivational Problems on Exponential and Logarithmic Functions

1. The function $p(t) = 3960(1.02)^t$ describes the population of Dilcue, North Dakota *t* years after it was founded.
   (a) Find the founding population.
   (b) At what annual rate has the population of Dilcue been growing?
   c. Solve the equation p(t) = 77218. What is the meaning of your answer? By the way, notice that this question asks you to *find an exponen*t.

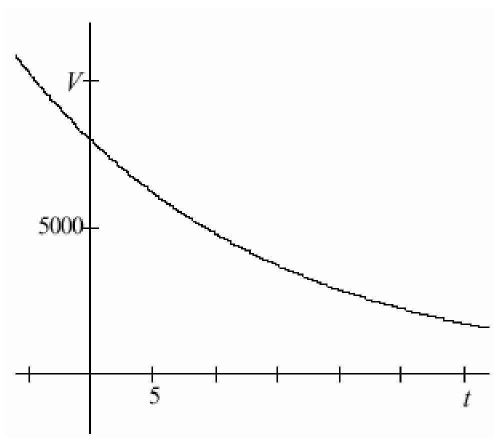
2. The equation at right shows part of the graph of $P(t) = 5280(1.024)^t$, a function that describes a small town whose population has been growing at an annual rate of 2.4 percent.



(a) What is P(0), and what is its meaning?
(b) Use the graph to estimate the solution of the equation P(t) = 10560.
(c) Calculate P(–30). What is the meaning of this number?
(d) Comment on the part of the graph that lies outside the borders of the illustration. How would it look if you could see it, and what does it mean?

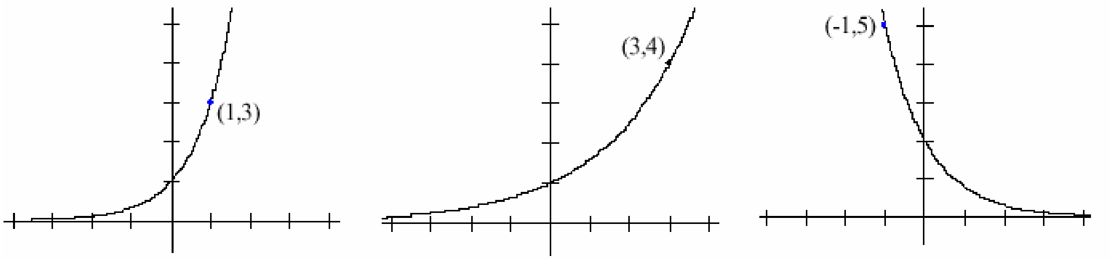
3. A helium-filled balloon is slowly deflating. During any 24-hour period, it loses 5 percent of the helium it had at the beginning of that period. Assume that the balloon held 8000 cc of helium at noon on Monday. How much helium did it contain 3 days later? 4.5 days later? 20 days later? *n* days later? 12 hours later? *k* hours later? Approximately how much time is needed for the balloon to lose half its helium? This time is called the *half-lif*e. Be as accurate as you can.

4. The equation at right shows part of the graph of *V t* (t) = 8000 (0.95). This function tells the story of a shrinking balloon that loses 5 percent of its helium each day.



(a) What is V(0), and what is its significance?
(b) Use the graph to estimate the t-value that solves the equation V(t) = 4000.
(c) Calculate V(–3). What does this value mean?
(d) Comment on the part of the graph that lies outside the borders of the illustration. How would it look if you could see it, and what does it mean?

5. Find a plausible equation $y = a \cdot b^x$ for each of the exponential graphs shown below:



6. Make up a context for the equation $f(x) = 5000(1.005)^x$. What would f(2000) represent in your context?

7. A constant monthly interest rate of 1.4% is equivalent to what annual interest rate?

8. Write each of the following numbers as a power of 10. You should not need a calculator.

(a) 1000 (b) 1000000 (c) 0.01 (d) 10 (e) $100\sqrt{10}$

(f) $\frac{1}{\sqrt[3]{100}}$

9. Rewrite the *logarithmic equation* $4 = \log 10000$ as an exponential equation.

(b) Rewrite the exponential equation $10^{3.03...} = 1997$.

10. Without using your calculator, solve each of the following equations:

(a) $8^x = 32$ b) $27^x = 243$ (c) $1000^x = 100000$

Explain why all three equations have the same solution.

11. Using your calculator for confirmation, and remembering that *logarithms are exponent*s, explain why it is predictable

(a) that log 64 is three times log 4;

(b) that log 12 is the sum of log 3 and log 4;

(c) that log 0.02 and log 50 differ only in sign.

12. For each description of an exponential function $f(x) = k \cdot b^x$ find *k* and b:

(a) f(0) = 3 and f(1) = 12 (b) f(0) = 4 and f(2) = 1

13. Given that $10^{.301} = 2$ and $10^{0.477} = 3$, you should not need your calculator to solve

(a) $10^x = 6$ (b) $10^x = 8$ (c) $10^x = \frac{2}{3}$ (d) $10^x = 1$

14. Given that 0.301 = log 2, and that 0.477 = log 3, you should not need a calculator to evaluate
(a) log 6; (b) log 8; (c) log (2/3); (d) log 1

15. Given that $m = \log a$ and $n = \log b$,
(a) express *a* as a power of 10, and express *b* as a power of 10;
(b) use your knowledge of exponents to express *ab* as a power of 10;
(b) conclude that log(*a*b) = log(a) + log(b).

16. (Continuation) Justify the following:

a. $\log_b x^r = r \log_b x$ b. $\log_b(\frac{x}{y}) = \log_b x - \log_b y$

17. Another approach to solving an equation like $5^x = 20$ is to *calculate base-10 logarithms of both sides of the equation.* First justify the equation $x \log 5 = \log 20$ then obtain the desired answer in the form $x = \frac{\log 20}{\log 5}$. Evaluate this expression. Notice that $\log_5 20 = \frac{\log 20}{\log 5}$.

18. Asked to "simplify" $x = \frac{\log 20}{\log 5}$, a student replied "log 4". What do you think of this answer?

19. The function $f(x) = 31416(1.24)^x$ describes the number of mold spores found growing on a pumpkin pie *x* days after the mold was discovered.
(a) How many spores were on the pie when the mold was first discovered?
(b) How many spores were on the pie two days *before* the mold was discovered?
(c) What is the daily rate of growth of this population?
(d) What is the *hourly* rate of growth?
(e) Describe the spore count on the same pie by the function G(x), where *x* counts the number of hours since the mold was discovered on the pie.

20. It has been discovered that the logarithms of two quantities *H* and *k* are related by the equation $\log H = 1.48 - 2.5 \log k$. Relate these two quantities by an equation that makes no reference to logarithms.