Motivational Problems on Sequences
Precalculus
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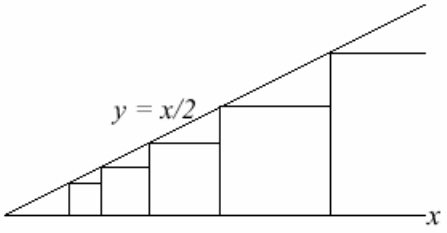
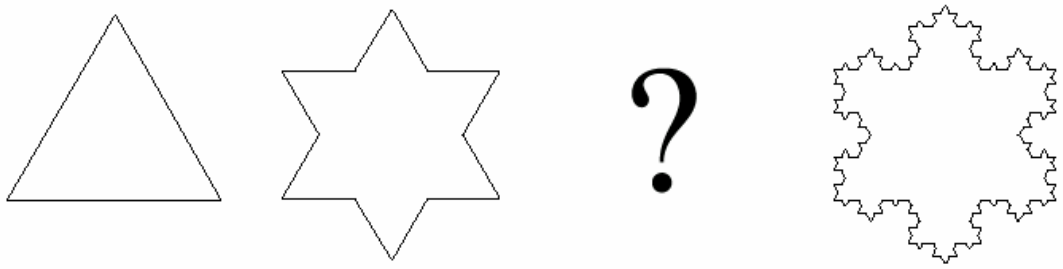
1. The figure at right shows a sequence of squares inscribed in the angle formed by the line $y = x/2$ and the x-axis. Each square has two vertices on the x-axis and one on the line $y = x/2$, and neighboring squares share a vertex. The smallest square is 8 cm tall. How tall are the next four squares in the sequence? How tall is the *n*th square in the sequence?



2. A sequence is defined as a function whose domain is the set of positive integers. You generally list the range, in a list separated by commas. For example, the sequence defined by $s_n = n^2$ would be written as 1, 4, 9, 16, 25, ….. Write a sequence formula for the even integers. Write a sequence formula for the odd integers.

3. A *geometric sequence* is a list in which each term is obtained by multiplying its predecessor by a constant. For example, 81, 54, 36, 24, 16, . . . is geometric, with constant multiplier 2/3. The first term of this sequence is 81; what is the 40th term? the millionth term? the *nth* term? Check your formula for small values of n.

4. In 1904 Helge von Koch invented his *snowflak*e, which is probably the first published example of a *fractal.* It is the result of an endless sequence of stages: Stage 0 (the initial configuration) consists of an equilateral triangle, whose sides are 1 unit long. Stage 1 is obtained from stage 0 by replacing the *middle third* of each edge by a pair of segments, arranged so that a small equilateral triangle protrudes from that edge. In general, each stage is a polygon that is obtained by applying the middle-third construction to *every* edge of the preceding stage.
   (a) Stages 0, 1, and 3 are shown above. Make your own sketch of stage 2.
   (b) Stage 0 has three edges, and stage 1 has twelve. How many edges do stages 2 and 3 have? How many edges does stage *n* have?
   (c) Stage 1 has twelve vertices. How many vertices does stage *n* have?
   (d) How long is each edge of stage 1? of stage 2? of stage n?
   (e) What is the perimeter of stage 1? of stage 2? of stage n?
   (f) Does the completed snowflake have finite perimeter? Explain.
   (g) Is the area enclosed by the completed snowflake finite? Explain.



5. An arithmetic Sequence is defined to be a list of numbers that is obtained by adding a constant to its predecessor. If the numbers in the sequence are decreasing what does that tell you about the constant? For example, 81, 69, 57, 45, 33…is an arithmetic sequence. What is the tenth term? The millionth term? The *nth* term?

6. Write a compact expression that represents the sequence in terms of n (where n is the number of that term. For example the first term has n=1, the second term has n=2)

    a. 1,4,7,…
    b. 8, 6, 4, 2….
    c. 8, 4, 2, 1, ….
    d. 24, -12, 6, -3….

7. How many terms do you think there are in the arithmetic sequence 18, 24, ….., 336?

8. If the first term of a geometric sequence is 2, the second is $\sqrt{8}$ what is the thirteenth term?