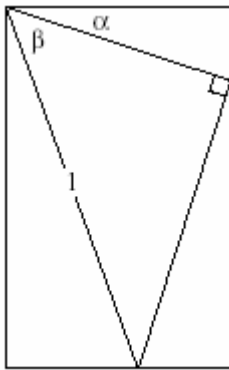


Motivational Problems on Sum and Difference Trig Identities



1. The rectangle shown to the left has been formed by fitting together four right triangles. As marked, the sizes of two of the angles are  $\alpha$  and  $\beta$ , and the length of one segment is 1. Find the two unmarked angles whose sizes are  $\alpha$  and  $\alpha + \beta$ .
2. Once you know where  $\alpha + \beta$  is in the picture, label the side of the rectangle that could be written as  $\sin(\alpha + \beta)$ .
3. Consider the right triangle in the center with hypotenuse 1. How could you write expressions for the other two sides of that triangle?
4. (Continuation) Using those sides from #3, write expressions for the other parts of the rectangle.
5. Because rectangles have congruent opposite sides, come up with an equivalent expression for  $\sin(\alpha + \beta)$ .
6. You have just found a trigonometric identity for the sine of a sum of two angles. When asked to find an expression that is equivalent to  $\cos(a+b)$ , a student responded " $\cos a + \cos b$ ". What do you think of this answer, and why?
7. Explain why the equation  $\tan \theta = 2$  has solutions, but the equation  $\sin \theta = 2$  does not.
8. Recall your work in problems 1-5. See if you can find an identity for the  $\cos(\alpha + \beta)$  using the same picture. Good Luck!
9. You need to find the  $\sin(55)$  degrees. You just spilled soda on your calculator and the "4" and "5" buttons on your calculator are stuck. You can, however, find the sine and cosine of 10 degrees, and you know, from Algebra 2 Trig, the sine and cosine of 45. With just this information, how could you find the  $\sin(55)$ ?
10. If you are given that  $\tan \theta = \frac{-3}{4}$ , what are  $\sin \theta$  and  $\cos \theta$ ?
10. If  $\alpha$  and  $\beta$  are angles in the first quadrant, draw a picture that represents the angle  $\alpha - \beta$ . If these angles represent angles for points on the unit circle, you now have a triangle with 2 sides that are one unit long and an angle in between them is  $\alpha - \beta$ . Using the law of Cosines, come up with an expression the distance AB (the third side of the triangle).
11. Let the point A =  $(\cos \alpha, \sin \alpha)$  and B =  $(\cos \beta, \sin \beta)$ . Using the distance formula, find an expression for the distance AB. Notice that this distance should be the same as the third side of the triangle you came up with in #10.
12. You now have 2 expressions for the distance AB. Set them equal to each and solve for  $\cos(\alpha - \beta)$  to come up with an identity for  $\cos(\alpha - \beta)$ .

13. You are given the expression  $\sin \frac{5\pi}{12} \cos \frac{\pi}{12} + \cos \frac{5\pi}{12} \sin \frac{\pi}{12}$ , and your calculator doesn't work. How could you still find the exact value for this expression?
14. Derive the identity for  $\sin(\alpha - \beta)$  yourself by substituting  $-\beta$  in for  $\beta$  in the original identity for  $\sin(\alpha + \beta)$ . You can do it!
15. You need to find the  $\sin(-15)$  and don't have your calculator. How could you use one of the 4 trigonometric identities for sum or difference to find an exact value for  $\sin(-15)$ .