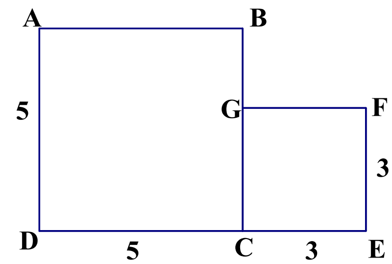
1. A 5×5 square and a 3×3 square can be cut into pieces that will fit together to form a third square.
   (a) Find the length of a side of the third square.
   (b) In the diagram at right, mark *P* on segment *DC* so that *PD* = 3, then draw segments *PA* and *PF*. Calculate the lengths of these segments.
   (c) Segments *PA* and *PF* divide the squares into pieces. Arrange the pieces to form the third square.



2. (Continuation) Change the sizes of the squares to *AD* = 8 and *EF* = 4, and redraw the diagram. Where should point *P* be marked this time? Form the third square again.

3. (Continuation) Will the preceding method *always* produce pieces that form a new square? If your answer is *yes*, prepare a written explanation. If your answer is *no*, provide a counterexample – two specific squares that can *not* be converted to a single square.

4. *Some terminology*: In a right triangle, the *legs* are the sides adjacent to the right angle. The *hypotenuse* is the side opposite to the right angle. Given the two points A(3, 7) and B(5, 2) find C so that triangle ABC is a right triangle with the right angle at C. How long are legs? How long is the hypotenuse?

5. Let $A = (0, 0)$, $B = (7, 1)$, $C = (12, 6)$, and $D = (5, 5)$. Plot these points and connect the dots to form the *quadrilateral ABCD*. Verify that all four sides have the same length. Such a figure is called *equilateral*.

6. The main use of the Pythagorean Theorem is to find distances. Originally (6*th* century BC), however, it was regarded as a statement about *areas*. Explain this interpretation.

7. The length of a rectangle is $(3x – 4)$ and the width is $(2x + 1)$. Find the perimeter and area of this rectangle.

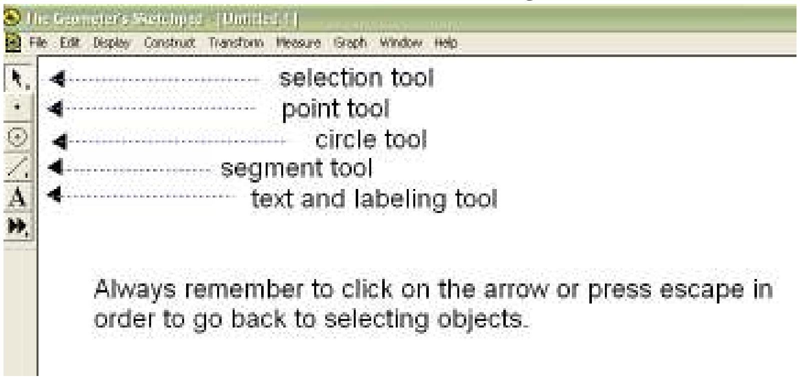
8. Using your graphing calculator**,** press the y = button and enter the parabola $y = 2x^2 - 133x + 2$.
   (a) What do you see on your screen?
   (b) Press the Window button (next to y =), and enter Xmin = -50, Xmax = 50, Ymin = -500, Ymax = 500. Press Graph. Now, what do you see?
   (c) Find a window where you can see the bottom of the curve. Which window did you pick? Present your answer as [Xmin, Xmax], [Ymin, Ymax].

9. *Factor:* $x^2 - 5x + 6$

10. If the hypotenuse of a right triangle is 12 and one of the legs is 4 express the length of the other leg in simplest radical form.

GSP Lab #1

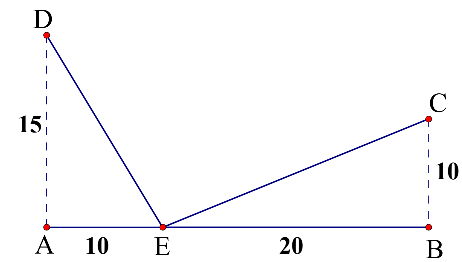
Open Geometer's Sketchpad on your computer and observe the toolbox on the left-hand side of your screen. The first five tools are the most important for your work in this course. The 2nd, 3rd and 4th tools are also accessible through the construct menu.



If you have never used a dynamic software program before, you should be aware of the difference between properties and objects that are "constructed" (using either a tool or a CONSTRUCT command) rather than "drawn" freehand with a tool. To observe this difference, complete the following activity:

A. Open a new sketch in GSP.
B. Click on the segment tool from the toolbox on the left-hand side of the screen.
C. Draw a right triangle by constructing a segment with this segment tool, being sure to connect the endpoints of the segments. The point will highlight itself when you are right on top of it with the segment construction tool.
D. Press escape (ESC), or click on the selection tool again and click on any endpoint of a segment and move the triangle around. You will notice that the original form you drew is no longer on the sketch. However, EDIT→undo will get you back to your original drawing.
E. Now you will construct a right triangle. Choose the segment tool from the left-hand side toolbox and hold down the shift key while you draw a horizontal segment.
F. Press ESC or click on the selection tool again and select both the horizontal segment and its left-hand endpoint.
G. Now choose CONSTRUCT→perpendicular line. A line through the endpoint and perpendicular to the segment should be constructed.
H. Select the segment tool from the toolbox and click on the left-hand endpoint where the perpendicular line was constructed and draw a line segment vertically on top of the line. Don't forget to return to the selection tool after you have constructed the perpendicular.
I. Select the line (not the line segment) and choose DISPLAY→Hide Perpendicular Line.
J. Finally, with the segment tool, draw in the hypotenuse of this right triangle from endpoint to endpoint of the two perpendicular sides. Be sure not to click until the point is highlighted by the tool.
K. Drag the vertices of this right triangle. How is this right triangle different from the one you drew in part C? Come to class prepared to discuss the differences.

1. In the diagram, *AEB* is straight and angles *A* and *B* are right. Calculate the total distance *DE* + *EC*.



2. (Continuation) If *AE* = 15 and *EB* = 15 instead, would *DE* + *EC* be the same?

3. (Continuation) You have seen that *DE* + *EC* is dependent on the length of *AE*. Let *x* represent *AE* (and 30 − *x* for *EB*), write a formula for *DE* + *EC*. Enter this formula into your calculator, graph it, and find the value of *x* that produces the *shortest* path from *D* to *C* through *E*. Draw an accurate picture of *this* path, and make a conjecture about *angles AED* and *BEC*. Use your protractor to test your conjecture.

4. Two different points on the line $y = 2$ are each exactly 13 units from the point (7*,* 14). Draw a picture of this situation, and then find the coordinates of these points.

5. The general notation in geometry is that points are labeled with capital letters and coordinates are defined with lowercase letters. Given the two points $A(x_1, y_1)$ and $B(x_2, y_2)$ what do the subscripts on *x* and *y* represent? If triangle ABC is right triangle with C being the right angle, find expressions for all three sides.

6. To find the intersection of two lines using your calculator, first graph the lines and make sure that you can see the intersection point in the window. If you can't, be sure to change the window accordingly. Then bring up the CALC menu (2nd trace) and choose intersect. Using the arrow keys, move the cursor along one of the lines close to the point of intersection and press ENTER. The calculator will prompt you to then press ENTER on the other intersecting line. Then, read the coordinates of the intersection point from the bottom of your screen.
   Find the intersection of $4x + 5y = 8$ and $y = 0.6x – 4$.

7. Give an example of a point that is the same distance from (3*,* 0) as it is from (7*,* 0). Find lots of examples. Describe the configuration of all such points. In particular, how does this configuration relate to the two given points?

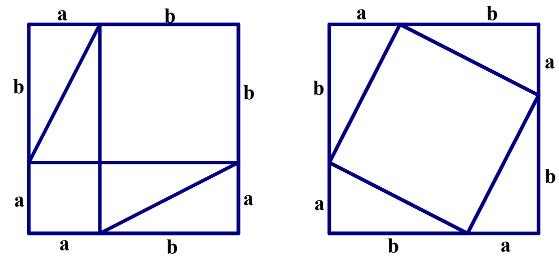
8. Verify that the hexagon formed by *A* = (0*,* 0), *B* = (2*,* 1), *C* = (3*,* 3), *D* = (2*,* 5), *E* = (0*,* 4), and *F* = (-1*,* 2) is equilateral. Is it also *equiangular*?

9. Draw a 20-by-20 square *ABCD*. Mark *P* on *AB* so that *AP* = 8, *Q* on *BC* so that *BQ* = 5, *R* on *CD* so that *CR* = 8, and *S* on *DA* so that *DS* = 5. Find the lengths of the sides of quadrilateral *PQRS*. Is there anything special about this quadrilateral? Explain.

10. Given the two points A(-2, 1) and B(4, 7) describe two different methods to find the distance between A and B. Which method do you prefer?

1. Verify that $P = (1, -1)$ is the same distance from $A = (5, 1)$ as it is from $B = (-1, 3)$. It is customary to say that P is *equidistant* from A and B. Find three more points that are equidistant from A and B. By the way, to "find" a point means to find its *coordinates*. Can points equidistant from A and B be found in every *quadrant*?

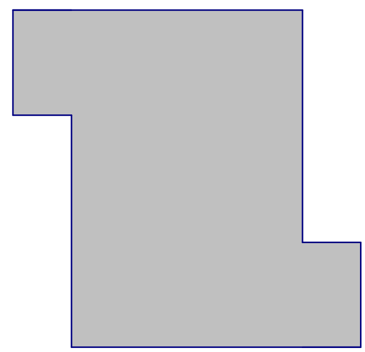
2. The two-part diagram at right, which shows two different dissections of the same square, was designed to help *prove* the Pythagorean Theorem. Provide the missing details.



3. Find both points on the line $y = 3$ that are 10 units from $(3, -3)$

4. On a number line, where is $\frac{1}{2}(p + q)$ in relation to p and q?

5. Some terminology: Figures that have exactly the same shape and size are called *congruent*. Dissect the region shown at right into two congruent parts. How many different ways of doing this can you find?



6. Let $A = (2, 4)$, $B = (4, 5)$, $C = (6, 1)$, $T = (7, 3)$, $U = (9, 4)$, and $V = (11, 0)$. Triangles ABC and TUV are specially related to each other. Make calculations to clarify this statement, and write a few words to describe what you discover.

GSP Lab #2

In this lab, you will become familiar with the CONSTRUCT menu.

A. Open a new sketch on GSP.
B. Using the point tool on the toolbox to the left, draw two points.
C. Label the points A and B by using the Labeling tool to click on the points themselves.
D. Using the selection tool, select point A, then select point B. Choose CONSTRUCT→Circle by Center and Point. Notice A is the center of a circle and point B is on the circle.
E. Answer all of the following questions in a text window on your sketch by double-clicking with the text (labeling) tool.
F. Try selecting B then A. Compare your results to part C. What is the same? What is different?
G. What choices are in the CONSTRUCT menu if you have only one point selected?
H. What choices are in this menu if you have two points selected? Try some of them.
I. What choices are in the CONSTRUCT menu if you have three points selected? Try some of them.
J. What choices are in the CONSTRUCT menu if you have one point and a segment selected? Try some of them.
K. What choices are in this menu if you have a segment selected? Try some of them.

1. If you were writing a geometry book, and you had to define a mathematical figure called a *kite*, how would you word your definition?

2. A triangle that has two sides of equal length is called *isosceles*. Make up an example of an isosceles triangle, one of whose vertices is (3, 5). Give the coordinates of the other two vertices. If you can, find a triangle that does not have any horizontal or vertical sides.

3. Instead of walking along two sides of a rectangular field, Fran took a shortcut along the diagonal.
   (a) Let a be the short side, b be the long side, and c her path length. Write an equation that relates these variables.
   (b) By taking the shortcut along the diagonal, Fran can save a distance equal to half of the length of the longer side. Write an equation that describes this relationship.
   (c) Find the length of the long side of the field, given that the length of the short side is 156 meters.

GSP Lab #3

Using GSP, you can easily rotate objects when you know a center of rotation and an angle. In a sketch, draw a small triangle and double click on one of the vertices. This will mark that vertex as a center of rotation. "Lasso" (drag a box around) the triangle using the selection tool. Choose TRANSFORM→Rotate and the pop-up menu will ask you for an angle of rotation. You will notice that positive angles will rotate counterclockwise, while negative angles will rotate clockwise. Try creating a point not on the triangle and mark that as center. What changes when you move the center of rotation? What stays the same?

4. Let $A = (1, 5)$ and $B = (3, -1)$. Verify that $P = (8, 4)$ is equidistant from A and B. Find at least two more points that are equidistant from A and B. Describe all such points.

5. Solve for x: $\sqrt{x+1} = \sqrt{2x-3}$. Hint: you can square both sides to eliminate the radical.

6. Find two points on the y-axis that are 9 units from (7, 5).

7. A *lattice point* is a point whose coordinates are *integers*. For example, (2, 3) is a lattice point, but (2.5, 3) is not. Find two lattice points that are 5 units apart but not form a horizontal or vertical line.

8. (Continuation) Find two lattice points that are exactly $\sqrt{13}$ units apart. Is it possible to find lattice points that are $\sqrt{15}$ units apart?

1. *Some terminology*: When two angles fit together to form a straight angle (a 180-degree angle, in other words), they are called *supplementary angles* and either angle is the *supplement* of the other. When an angle is the same size as its supplement (a 90-degree angle), it is called a *right angle*. When two angles fit together to form a right angle, they are called *complementary* angles and either angle is the *complement* of the other. Two lines that form a right angle are said to be *perpendicular*. Draw a diagram for each definition.

2. With what you have learned about constructions in GSP, try to **construct** a square. Come to class either with the completed sketch to share, or be prepared to demonstrate for the class your idea. Be careful not to simply **draw** the square.

3. Two iron rails, each 50 feet long, are laid end to end with no space between them. During the summer, the heat causes each rail to increase in length by 0.04 percent. Although this is a small increase, the lack of space at the joint makes the joint buckle upward. What distance upward will the joint be forced to rise? (Assume that each rail *remains straight*, and that the other ends of the rails are anchored.) Round your answer to the nearest hundredth.

4. Graph the lines $2x - y = 5$ and $x + 2y = -10$ on a piece of graph paper on the same set of axes. Using your protractor, measure the angle of intersection.

5. Given that $2x - 3y = 17$ and $4x + 3y = 7$, and without using paper, pencil, or calculator, find the value of x.

6. Blair is walking along the edge of her room toward a wall where an Emma bug is crawling along the crown molding (top edge of the wall). Assuming the bug does not change direction, will their paths ever cross? Are their paths *parallel*?

7. The point on segment AB that is equidistant from A and B is called the *midpoint* of AB. For each of the following, find coordinates for the midpoint of AB:
   (a) $A = (-1, 5)$ and $B = (5, -7)$ **(b)** $A = (m, n)$ and $B = (k, l)$

8. A unique line exists through any two points. In one form or another, this statement is a fundamental *postulate* of Euclidean geometry – accepted as true, without proof. Taking this for granted, then, what can be said about three points?

9. Using GSP, plot the points P(3, 5), Q(0, 0) and R(-5, 3). Measure angle PQR. Create the segments PQ and QR. Select segment PQ (not its endpoints) and choose MEASURE → slope. Do the same thing for segment QR. Make a conjecture about how these slopes are related. Verify by calculating the slopes by hand.

1. Write a formula for the distance from $A = (-1, 5)$ to $P = (x, y)$, and another formula for the distance from $P = (x, y)$ to $B = (5, 2)$. Then write an equation that says that P is equidistant from A and B. Simplify your equation to linear form.

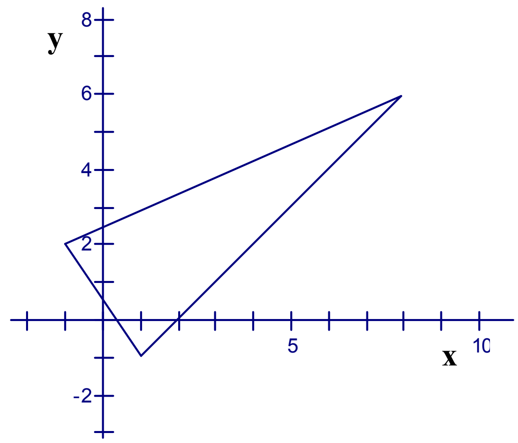
2. (Continuation) The line you just found is called the *perpendicular bisector of AB*. Verify this by calculating two slopes and one midpoint.

3. Is it possible to form a square whose area is 18 by connecting four lattice points? Explain.

4. Find the slope of the line through
(a) $(3, 1)$ and $(3 + 4t, 1 + 3t)$; **(b)** $(m - 5, n)$ and $(5 + m, n^2)$.

5. Is it possible for a line $ax + by = c$ to lack a y-intercept? To lack an x-intercept? Explain.

6. Factor: **(a)** $x^2 - 16$; **(b)** $x^2 + 8x + 16$; **(c)** $x^2 + 6x - 16$



7. Find the point of intersection of the lines $3x + 2y = 1$ and $-x + y = -2$.

8. (Continuation) The sides of the triangle at right are formed by the graphs of $3x + 2y = 1$, $y = x - 2$, and $-4x + 9y = 22$. Is the triangle isosceles? How do you know?

9. Consider the linear equation $y = 3.5(x - 1.3) + 2$.
(a) What is the slope of this line?
(b) What is the value of y when $x = 1.3$?
(c) This equation is written in *point-slope* form. Explain the terminology.
(d) Use your calculator to graph this line.
(e) Find an equation for the line through $(4.2, -2.5)$ that is parallel to this line. Leave your answer in point-slope form.
(f) Describe how you would graph by hand a line that has slope -2 and that goes through the point $(-7, 3)$.

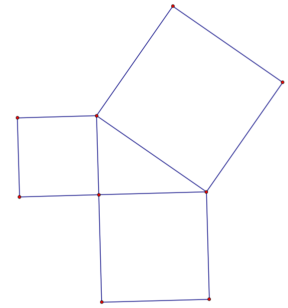
10. For each of the following questions, fill in the blank with always true (A), never true (N), or sometimes true (S). Please write a few sentences explaining your choice.
(a) Two parallel lines are ________ coplanar.

(b) Two lines that are not coplanar _______ intersect.

(c) Two lines parallel to the same plane are ________ parallel to each other.

(d) Two lines parallel to a third line are ________ parallel to each other.

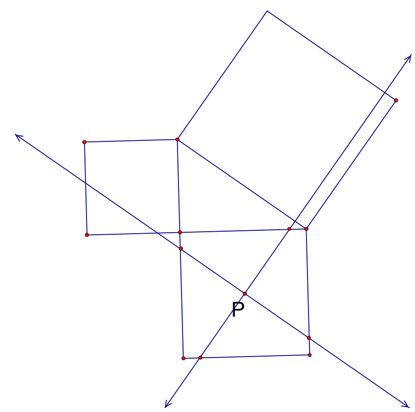
(e) Two lines perpendicular to a third line are ________ perpendicular to each other.



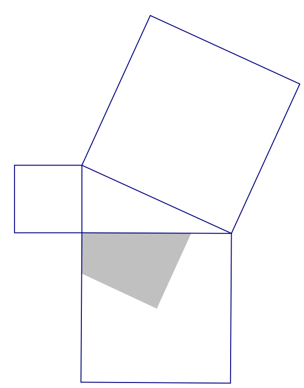
GSP Lab #4

In the following lab, you will prove the Pythagorean Theorem by "dissection" – explain this terminology after you have completed the lab.

A. Open a new sketch with GSP.
B. Construct a right triangle (with directions E-J from Lab #1)
C. Construct squares (by any method you wish) extending from all three sides of the right triangle. Your sketch should look like this:
D. This will geometrically represent the a^2, b^2 and c^2 in the Pythagorean Theorem that most students know algebraically.
E. You will now make some pieces to put together as a puzzle.
F. Construct the diagonals of the square that is connected to the longer leg of the right triangle.
G. Construct the intersection of those diagonals using the Arrow (Selection Tool). Simply click on the place where the intersection point would appear. (Hint: do not use the point tool for this, just the Arrow).
H. Hide the two diagonals, but not the center point. Using the Label Tool, label that point P. By clicking on the point with the Label Tool, the automatic label will appear. To change the label, double-click on the actual label and a pop-up window will allow you to change the letter of the label.



I. Select the point P and the hypotenuse of the original right triangle. Select CONSRUCT→ Parallel Line.
J. Select the point P and the hypotenuse of the original right triangle. Select CONSTRUCT → Perpendicular line.
K. Using the selection tool, click on the 4 points of intersection between the two lines you just constructed and the square connected to the longer leg of the original right triangle. See diagram.
L. Hide the two lines.
M. Now you will construct 5 polygon interiors to use as the puzzle pieces. First select the four points that will be the vertices of one quadrilateral. With the four points selected, select CONSTUCT→Polygon Interior.

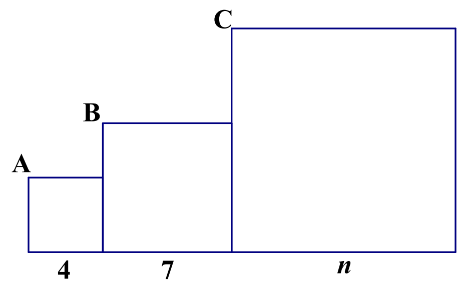


N. Repeat this for all 4 of the quadrilaterals in the square connected to the longer leg, and then for the whole smaller square. We recommend making each polygon interior different colors by choosing DISPAY→color.
O. All points in the diagram need to be hidden now, so select the point tool and go to EDIT→Select all points, then choose DISPLAY→Hide points.
P. One by one, select each of the quadrilateral and choose EDIT→copy and then paste. You will use these 5 non-dynamic pieces to fit into the largest square coming off the hypotenuse.
Q. Why does this prove the Pythagorean Theorem?

1. A slope can be considered to be a *rate*. Explain this interpretation and give an example.

2. Given the points $A = (-2, 7)$ and $B = (3, 3)$, find two points P that are on the perpendicular bisector of AB. In each case, what can be said about the triangle PAB?

3. Explain the difference between a line that has *undefined slope* and a line whose slope is zero.



4. Three squares are placed next to each other as shown. The vertices A, B, and C are *collinear*. Find the dimension n.

5. (Continuation) Replace the lengths 4 and 7 by m and k, respectively. Express k in terms of m and n.

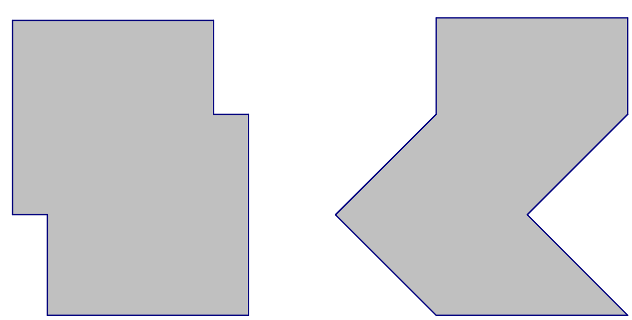
6. A five-foot tall Emma student casts a shadow that is 40 feet long while standing 200 feet from a streetlight. How high above the ground is the lamp?

7. (Continuation) How far from the streetlight should the student stand in order to cast a shadow that is exactly as long as the student is tall?

8. An airplane 27000 feet above the ground begins descending at the rate of 1500 feet per minute. Assuming the plane continues at the same rate of descent, how long will it be before it is on the ground?

9. (Continuation) Graph the line $y = 27000 - 1500x$, using an appropriate window on your calculator. With the preceding problem in mind, explain the significance of the slope of this line and its two intercepts.

10. In a dream, Blair is confined to a coordinate plane, moving along a line with a constant speed. Blair's position at 4 am is (2, 5) and at 6 am it is (6, 3). What is Blair's position at 8:15 am when the alarm goes off?



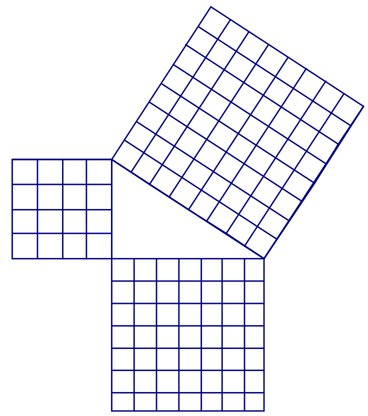
11. Find a way to show that points $A = (-4, -1)$, $B = (4, 3)$, and $C = (8, 5)$ are collinear.

12. Find as many ways as you can to dissect each figure at right into two congruent parts.

13. One of the legs of a right triangle is 12 units long. The other leg is b units long and the hypotenuse c units long, where b and c are both integers. Find two possible values for b and c. Hint: both sides of the equation $c^2 - b^2 = 144$ can be factored.

1. An airplane is flying at 36000 feet directly above Lincoln, Nebraska. A little later the plane is flying at 28000 feet directly above Des Moines, Iowa, which is 160 miles from Lincoln. Assuming a constant rate of descent, predict how far from Des Moines the airplane will be when it lands.

2. What is wrong with the figure shown at right?



3. Suppose that numbers *a*, *b*, and *c* fit the equation $a^2 + b^2 = c^2$, with $a = b$. Express *c* in terms of *a*. Draw a good picture of such a triangle. What can be said about its angles?

4. Golf balls cost \$0.90 each at Emma's Club, which has an annual \$25 membership fee. At Wendy & Marilyn's sporting goods store, the price is \$1.35 per ball for the same brand. Where you buy your golf balls depends on how many you wish to buy. Explain, and illustrate your reasoning by drawing a graph.

5. Draw the following segments. What do they have in common?
   (a) from (3, -1) to (10, 3); **(b)** from (1.3, 0.8) to (8.3, 4.8); **(c)** from ($\sqrt{3}$, $\sqrt{2}$) to ($7+\sqrt{3}$, $4+\sqrt{2}$).

6. (Continuation) The above segments all have the *same* length and the *same* direction. Each represents the *vector* [7, 4]. The horizontal *component* of the vector is positive 7 and the vertical component is positive 4.
   (a) Find another example of two points that represent this vector. The initial point of your segment is called the *tail* of the vector, and the final point is called the *head*.
   (b) Which of the following segments represents vector [7, 4]? From (-2, -3) to (5, -1); from (-3, -2) to (11, 6); from (10, 5) to (3, 1); from (-7, -4) to (0, 0).

7. Using GSP, you can facilitate visualization of vector translations. In a sketch, draw a line segment. Then highlight the endpoints of the segment and choose TRANSFORM→Mark Vector. You should see an animation showing you how the computer is remembering the vector you marked. Now draw a small triangle somewhere on the sketch and highlight the whole triangle. Choose TRANSFORM→Translate and be sure that the Marked vector option is chosen. You will notice that GSP shows you a "ghost" of the image of the triangle, and you should click the Translate button. You can also explicitly tell the computer which vector to use if you choose Rectangular as the translation vector. Feel free to play with this function.

8. Given the line $y = \frac{3}{4}(x+3) - 2$ and the point (9, 2). Using point-slope form, write equations for the lines parallel and perpendicular to this line through the given point.

9. Let $P = (a, b)$, $Q = (0, 0)$, and $R = (-b, a)$, where *a* and *b* are positive numbers. Prove that angle *PQR* is right, by introducing two congruent right triangles into your diagram. Verify that the slope of *segment QP* is the *negative reciprocal* of the slope of segment *QR*.

1. The perimeter of an isosceles right triangle is 24 cm. How long are its sides?

2. Show that the triangle formed by the lines $y = 2x - 7$, $x + 2y = 16$, and $3x + y = 13$ is isosceles. Show also that the lengths of the sides of this triangle fit the Pythagorean equation. Can you identify the right angle just by looking at the equations?

3. A triangle has vertices $A = (1, 2)$, $B = (3, -5)$, and $C = (6, 1)$. Triangle $A'B'C'$ is obtained by *sliding* triangle ABC 5 units to the right (in the positive x-direction, in other words) and 3 units up (in the positive y-direction). It is also customary to say that vector [5, 3] has been used to *translate* triangle ABC. What are the coordinates of A', B', and C'? By the way, "A prime" is the usual way of reading A'.

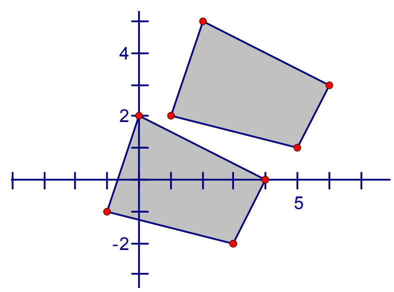
4. (Continuation) After triangle ABC has been translated h units in the x-direction and k units in the y-direction, vertex A lands at (-3, 7). Where do vertices B and C land?

5. A clock takes 3 seconds to chime at 3 pm, how long does it take to chime at 6 pm? Hint: There are pauses between the chimes.

6. Using GSP, plot the points $A = (-5, 0)$, $B = (5, 0)$, and $C = (2, 6)$; let $K = (5, -2)$, $L = (13, 4)$, and $M = (7, 7)$. Find the lengths of each side and the measure of each angle of the triangles. It is customary to call two triangles *congruent* when all corresponding sides and angles are the same.

7. (Continuation) Are the triangles related by a vector translation? Why?

8. A triangular plot of land has boundary lines of 45 meters, 60 meters, and 70 meters. The 60 meter boundary runs north-south. Is there a boundary line for the property that runs due east-west?



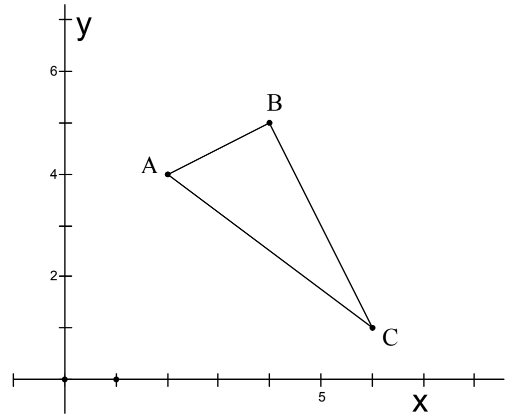
9. Let $A = (1, 2)$, $B = (5, 1)$, $C = (6, 3)$, and $D = (2, 5)$. Let $P = (-1, -1)$, $Q = (3, -2)$, $R = (4, 0)$, and $S = (0, 2)$. Use a vector to describe how quadrilateral $ABCD$ is related to quadrilateral $PQRS$.

10. Let $K = (3, 8)$, $L = (7, 5)$, and $M = (4, 1)$. Find coordinates for the vertices of the triangle that is obtained by using the vector [2, -5] to slide triangle KLM. How *far* does each vertex slide?

11. The *length of a vector* is defined as the hypotenuse of the right triangle created by its *components*. The horizontal component of the vector [-1, 7] is -1 and the vertical component is 7. What is the length of the vector [-1, 7]? What is the length of vector [a, b]? Some notation: the length of a vector is written as $\|[a, b]\|$.

12. Find all points on the y-axis that are twice as far from (-5, 0) as they are from (1, 0). Begin by making a drawing and estimating. Find all such points on the x-axis. In each case, how many points did you find? How do you know that you have found them all?

1. Let $A = (2, 4)$, $B = (4, 5)$, and $C = (6, 1)$. Triangle ABC is shown at right. Draw three new triangles as follows: **(a)** ΔPQR has $P = (11, 1)$, $Q = (10, -1)$, and $R = (6, 1)$; **(b)** ΔKLM has $K = (8, 10)$, $L = (7, 8)$, and $M = (11, 6)$; **(c)** ΔTUV has $T = (-2, 6)$, $U = (0, 5)$, and $V = (2, 9)$. These triangles are not obtained from ABC by applying a vector translation. Instead, each of the appropriate transformations is described by one of the suggestive names *reflection*, *rotation*, or *glide-reflection*. Decide which is which, with justification.



2. In baseball, the bases are placed at the corners of a square whose sides are 90 feet long. Home plate and second base are at opposite corners. How far is it from home plate to second base to two decimal places?

3. The juniors are jealous of the seniors and they want to copy the senior triangle onto the lacrosse field. They have a limited amount of time so they measure one of the sides and create a congruent segment on the field. If they do not do any more measurements does this guarantee that the junior triangle will be congruent to the senior triangle? Sketch a diagram of this scenario. Another group measured only one angle and created a congruent angle on the field. If they do not do any more measurements does this guarantee that the junior triangle will be congruent to the senior triangle? Sketch a diagram.

4. Give the components of the vector whose length is 10 and that points in the opposite direction of [-4, 3].

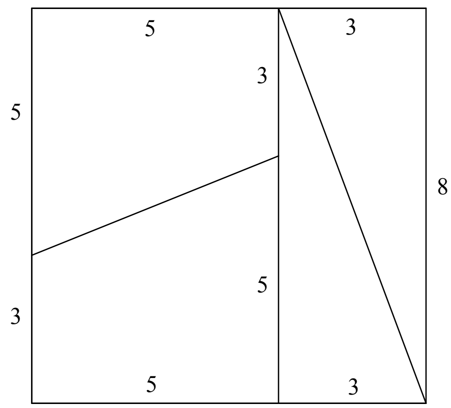
5. A 9-by-12 rectangular picture is framed by a border of uniform width. Given that the combined area of picture plus frame is 180 square units, find the width of the border.

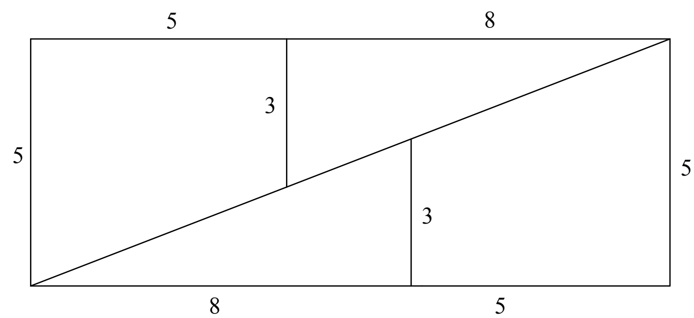
6. Let $A = (0, 0)$, $B = (2, -1)$, $C = (-1, 3)$, $P = (8, 2)$, $Q = (10, 3)$, and $R = (5, 3)$. Plot these points. Angles BAC and QPR should look like they are the same size. Find evidence to support this conclusion.

7. The juniors realize that copying a single measurement will not guarantee an exact copy of the senior triangle. They decide to try measuring two parts. What are the combinations of two corresponding parts that they could measure? Does the use of any of these pairs insure congruent triangles?

8. An equilateral quadrilateral is called a *rhombus*. A square is a simple example of a rhombus. Find a non-square rhombus whose *diagonals* and sides are *not* parallel to the rulings on your graph paper. Use coordinates to describe its vertices. Write a brief description of the process you used to find your example.

1. Compare the two figures shown below. Is there anything wrong with what you see? Write a few sentences justifying your answer.





2. A bug is initially at (-3, 7). Where is the bug after being displaced by vector [-7, 8]?

3. Plot points K = (0, 0), L = (7, -1), M = (9, 3), P = (6, 7), Q = (10, 5), and R = (1, 2). Show that the triangles KLM and RPQ are congruent. Show also that neither triangle is a vector translation of the other. Describe how one triangle has been transformed into the other.

4. What is the slope of the line $ax + by = c$? Find an equation for the line through the origin that is perpendicular to the line $ax + by = c$.

5. Realizing that one or two corresponding parts does not insure congruent triangles the juniors conjecture that they must use three parts to create a new junior triangle. Create a table of the possibilities. Which do you think will work and why?

6. (Continuation) Go to the class's webpage, and explore the sketches called Triangle Congruence Criteria. Do any of the sketches confirm or contradict your conjectures?

7. Choose a point P on the line $2x + 3y = 7$, and draw the vector [2, 3] with its tail at P and its head at Q. Confirm that the vector is perpendicular to the line. What is the distance from Q to the line? Repeat the preceding, with a different choice for point P.

8. Let A = (3, 2) and B = (7, -10). What is the displacement vector that moves point A onto point B? What vector moves B onto A?

9. Let $M = (a, b)$, $N = (c, d)$, $M' = (a + h, b + k)$, and $N' = (c + h, d + k)$. Use the distance formula to show that segments MN and $M'N'$ have the same length. Explain why this could be expected.

1. *Some terminology*: When the components of the vector [5, -7] are multiplied by a given number t, the result may be written either as [5t, -7t] or as t[5, -7]. This is called the *scalar multiple* of vector [5, -7] by the *scalar t*. Find components for the following scalar multiples:
   (a) [12, -3] by scalar 5 **(b)** $[\sqrt{5}, \sqrt{10}]$ by scalar $\sqrt{5}$
   (c) $\left[-\frac{3}{4}, \frac{2}{3}\right]$ by scalar $-\frac{1}{2}+\frac{2}{6}$ **(d)** [p, q] by scalar $\frac{1}{\sqrt{p^2q^2}}$

2. If two figures are congruent, then their parts *correspond*. In other words, each part of one figure has been matched with a definite part of the other figure. In the triangle *RPQ*, which angle corresponds to angle *M*? Which side corresponds to *KL*? In general, what can be said about corresponding parts of congruent figures? How might you confirm your hunch experimentally?

3. Given the vector [-5, 12], find the following vectors:
   (a) same direction, twice as long **(b)** same direction, length 1
   (c) opposite direction, length 10 **(d)** opposite direction, length c

4. Two of the sides of a right triangle have lengths $360\sqrt{2006}$ and $480\sqrt{2006}$. Find the possible lengths for the third side.

5. Find the lengths of the following vectors:
   (a) [3, 4] **(b)** 2006 [3, 4] **(c)** $\frac{2006}{5}$ [3, 4] **(d)** t[3, 4]
   (e) In terms of a and b: t[a, b]

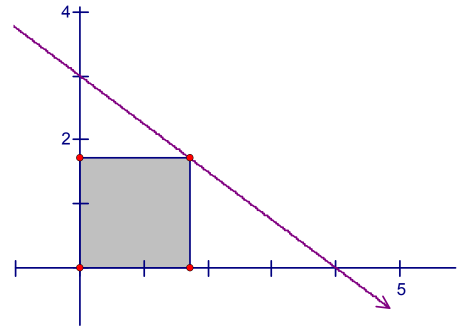
6. We know that |[3, 4]| = 5; how do we find a vector that points in the same direction and has length 1?

7. A triangle has six principal parts – three sides and three angles. The SSS criterion states that three of these items (the sides) determine the other three (the angles). What other combinations of three parts determine the remaining three? In other words, if the class is given three measurements with which to draw and cut out a triangle, which three measurements will guarantee that everyone's triangles will be congruent?

8. The vector that is defined by a segment *AB* is often denoted $\overrightarrow{AB}$. Given A(1, 1) and B(3, 5); **(a)** Use the midpoint formula to find the midpoint of $\overline{AB}$; **(b)** find the vector $\overrightarrow{AB}$ and multiply by $\frac{1}{2}$. **(c)** Translate A by $\frac{1}{2}\overrightarrow{AB}$. Did you expect this result?

9. Let $A = (1, 4)$, $B = (0, -9)$, $C = (7, 2)$, and $D = (6, 9)$. Prove that angles *DAB* and *DCB* are the same size. Can anything be said about the angles *ABC* and *ADC*?

1. The diagram at right shows the graph of $3x + 4y = 12$. The shaded figure is a square, three of whose vertices are on the coordinate axes. The fourth vertex is on the line. Find **(a)** the x- and y-intercepts of the line; **(b)** the length of a side of the square.



2. Plot the three points $P = (1, 3)$, $Q = (5, 6)$, and $R = (11.4, 10.8)$. Verify that $PQ = 5$, $QR = 8$, and $PR = 13$. What is special about these points?

3. Sidney calculated three distances of the collinear points A, B, and C. She reported them as $AB = 29$, $BC = 23$, and $AC = 54$. What do you think of Sidney's data, and why?

4. Find the number that is two thirds of the way **(a)** from -7 to 17; **(b)** from m to n.

5. The diagonal of a rectangle is 15 cm, and the perimeter is 38 cm. What is the area? It is possible to find the answer without finding the dimensions of the rectangle – try it.

6. After drawing the line $y = 2x - 1$ and marking the point $A = (-2, 7)$, Kendall is trying to decide which point on the line is closest to A. The point $P = (3, 5)$ looks promising. To check that P really is the point on $y = 2x - 1$ that is closest to A, what should Kendall do? Is P closest to A?

7. Let $K = (-2, 1)$ and $M = (3, 4)$. Find coordinates for the two points that divide segment KM into three congruent segments.

8. Find the components of a vector that is three fifths as long as [24, 7].

9. Let $A = (-5, 2)$ and $B = (19, 9)$. Find coordinates for the point P between A and B that is three fifths of the way from A to B. Find coordinates for the point Q between A and B that is three fifths of the way from B to A.

10. Given the points $K = (-2, 1)$ and $M = (3, 4)$, find coordinates for a point J that makes angle JKM a right angle.

11. When two lines intersect, four angles are formed. It is not hard to believe that the nonadjacent angles in this arrangement are congruent. If you had to prove this to a skeptic, what reasons would you offer?

12. One of the legs of a right triangle is twice as long as the other and the perimeter of the triangle is 28. Find the lengths of all three sides, to three decimal places.

13. A car traveling east at 45 miles per hour passes a certain intersection at 3 pm. Another car traveling north at 60 miles per hour passes the same intersection 25 minutes later. To the nearest minute, figure out when the cars are exactly 40 miles apart.

1. Find a point on the line $2x + y = 8$ that is equidistant from the coordinate axes. How many such points are there?

2. Let $A = (2, 9)$, $B = (6, 2)$, and $C = (10, 10)$. Verify that segments AB and AC have the same length. Measure angles ABC and ACB. On the basis of your work, propose a general statement that applies to any triangle that has two sides of equal length. Prove your assertion, which might be called the *Isosceles Triangle Theorem*.

3. A line goes through the points (2, 5) and (6, -1). Let P be the point on this line that is closest to the origin. Calculate the coordinates of P.

4. Given that $P = (-1, -1)$, $Q = (4, 3)$, $A = (1, 2)$, and $B = (7, k)$, find the value of k that makes the line AB **(a)** parallel to PQ; **(b)** perpendicular to PQ.

5. Let $A = (-6, -4)$, $B = (1, -1)$, $C = (0, -4)$, and $D = (-7, -7)$. Show that the opposite sides of quadrilateral $ABCD$ are parallel. Such a quadrilateral is called a *parallelogram*.

6. A line has an x-intercept at $(a, 0)$ and a y-intercept at $(0, b)$
(a) Write the equation of this line. **(b)** In terms of a and b, at what point does it intersect the line $y = x$?

7. The sides of a right triangle are $x - y$, x, and $x + y$, where x and y are positive numbers and $y < x$. Find the ratio of x to y.

8. Let $A = (0, 0)$, $B = (4, 2)$, and $C = (1, 3)$, find the exact size of angle CAB. Justify your answer without your protractor.

9. Let $A = (3, 2)$, $B = (1, 5)$, and $P = (x, y)$. Find x- and y-values that make ABP a right angle.

10. (Continuation) Describe the configuration of all such points P.

11. Find coordinates for the vertices of a *lattice rectangle* that is three times as long as it is wide with none of the sides horizontal.

12. Find components for the following vectors $\overrightarrow{AB}$:
(a) $A = (1, 2)$ and $B = (3, -7)$ **(b)** $A = (2, 3)$ and $B = (2 + 3t, 3 - 4t)$

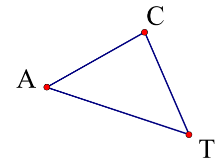
13. If $A = (-2, 5)$ and $B = (-3, 9)$, find components for the vector that points
(a) from A to B **(b)** from B to A

14. If M is the midpoint of segment AB, how are vectors $\overrightarrow{AM}$, $\overrightarrow{AB}$, $\overrightarrow{MB}$, and $\overrightarrow{BM}$ related?

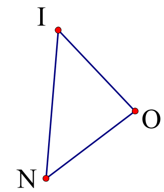
15. Show that the lines $3x - 4y = -8$, $x = 0$, $3x - 4y = 12$, and $x = 4$ form the sides of a rhombus.

1. Given the points E, W, and S with the property $EW = 5$, $WS = 7$, and $ES = 12$. What can be said about these three points? What would be true if ES is less than 12?

2. Describe a transformation that carries the triangle with vertices (0, 0), (13, 0), and (3, 2) onto the triangle with vertices (0, 0), (12, 5), and (2, 3). Where does your transformation send the point (6, 0)? If you cannot find the exact coordinates make your best guess.



3. Suppose that triangle ACT has been shown to be congruent to triangle ION, with vertices A, C, and T corresponding to vertices I, O, and N, respectively. It is customary to record this result by writing $\Delta ACT \cong \Delta ION$. Notice that corresponding vertices occupy corresponding positions in the equation. Let $B = (5, 2)$, $A = (-1, 3)$, $G = (-5,-2)$, $E = (1, -3)$, and $L = (0, 0)$. Using only these five labels, find as many pairs of congruent triangles as you can, and express the congruences accurately.



4. (Continuation) How many ways are there of arranging the six letters of $\Delta ACT \cong \Delta ION$ to express the two-triangle congruence?

5. What can be concluded about triangle ABC if it is given that
(a) $\Delta ABC \cong \Delta ACB$? **(b)** $\Delta ABC \cong \Delta BCA$?

6. Plot points $K = (-4, -3)$, $L = (-3, 4)$, $M = (-6, 3)$, $X = (0, -5)$, $Y = (6, -3)$, and $Z = (5, 0)$. Show that triangle KLM is congruent to triangle XZY. Describe a transformation that transforms KLM onto XZY. Where does this transformation send the point (-5, 0)?

7. Prove a property of the two acute angles in a right triangle.

8. Given $A = (6, 1)$, $B = (1, 3)$, and $C = (4, 3)$, find a lattice point P that makes the vectors $\overrightarrow{AB}$ and $\overrightarrow{CP}$ perpendicular.

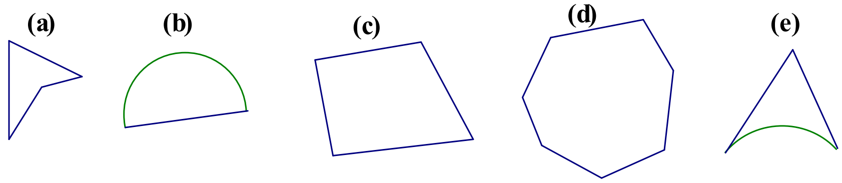
9. (Continuation) Write an equation of a line in point-slope form that describes the set of points P for which $\overrightarrow{AB}$ and $\overrightarrow{CP}$ are perpendicular.

10. Let $A = (0, 0)$, $B = (1, 2)$, $C = (6, 2)$, $D = (2, -1)$, and $E = (1, -3)$. Show that angle CAB is the same size as angle EAD. You may want to use Geometer's Sketchpad to help you solve this problem.

11. *Triangle Inequality Theorem:* What must be true about the three sides of a triangle for it to exist?

12. Which of these figures would you consider to be polygons? Justify your responses.

(a) **(b)** **(c)** **(d)** **(e)**



GSP Lab #5

Altitudes

A. Open GSP on the computer.
B. With the segment tool, draw yourself a random triangle and label it triangle ABC with the labeling tool.
C. Highlight in order, the points A, B, and C. Choose *Angle* from the MEASURE menu. The measurement of angle ABC should appear on your sketch.
D. Move a vertex of the triangle until this angle is acute.
E. Measure the other two angles and move the vertices until all three angles are acute as well.
F. Highlight vertex C and segment AB and choose *perpendicular line* from the CONSTRUCT menu.
G. With the selection tool (the arrow), click on the intersection point at the base of the altitude with side AB. A point should appear and you should label it point P.
H. Repeat step 6-7 for the other two vertices. In other words, create the altitudes from the other two vertices in the same manner.
I. Label the intersection point of the three altitudes R. This is called the **orthocenter** of the triangle.
J. Save this sketch as (Your name)'s Special Segments in a folder that is accessible to only you. For example mine would be called Ms. Schettino's Special Segments.

Answer the following questions:

1. Make a conjecture about the three altitudes of a triangle. What do you think is always true?
2. Test your conjecture by dragging one or more vertices around the sketch screen. What do you observe? Does this support your conjecture?
3. What do you observe when the triangle is obtuse?
4. What do you observe when the triangle is right?
5. What do you observe when the triangle is acute?

1. What is true about all of the points that lie on the perpendicular bisector of a segment?

2. Let $A = (0, 0)$, $B = (8, 1)$, $C = (5, -5)$, $P = (0, 3)$, $Q = (7, 7)$, and $R = (1, 10)$. Prove that angles ABC and PQR have the same size.

3. (Continuation) Let D be the point on segment AB that is exactly 3 units from B, and let T be the point on segment PQ that is exactly 3 units from Q. What evidence can you give for the congruence of triangles BCD and QRT?

4. The diagonals AC and BD of quadrilateral $ABCD$ intersect at O. Given the information $AO = BO$ and $CO = DO$, what can you deduce about the lengths of the sides of the quadrilateral? Prove your response.

1. An *altitude* of a triangle is a segment that joins one of the three vertices to a point on the line that contains the opposite side, the intersection being *perpendicular*. For example, consider the triangle whose vertices are $A = (0, 0)$, $B = (8, 0)$, and $C = (4, 12)$.
(a) Find the length of the altitude from C to side AB.
(b) Find an equation for the line that contains the altitude from A to side BC.
(c) Find an equation for the line BC.
(d) Find coordinates for the point where the altitude from A meets side BC.
(e) Find the length of the altitude from A to side BC.
(f) As a check on your work, calculate BC and multiply it by your answer to part (e). You should be able to predict the result.
(g) It is possible to deduce the length of the altitude from B to side AC from what you have already calculated. Show how.

2. Prove that one of the diagonals of a kite is bisected by the other.

3. Find a point on the line $x + 2y = 8$ that is equidistant from the points $(3, 8)$ and $(9, 6)$.

4. If a quadrilateral is equilateral, its diagonals are perpendicular. True or false? Why?

5. Make up a geometry problem to go with the equation $x + 3x + x\sqrt{10} = 42$.

6. Let $A = (-2, 3)$, $B = (6, 7)$, and $C = (-1, 6)$.
(a) Find an equation for the perpendicular bisector of AB.
(b) Find an equation for the perpendicular bisector of BC.
(c) Find coordinates for a point K that is equidistant from A, B, and C.

7. A 25 foot ladder is placed against a building. The bottom of the ladder is 7 feet from the building. If the top of the ladder slips down by 4 feet, by how many feet will the bottom slide out?

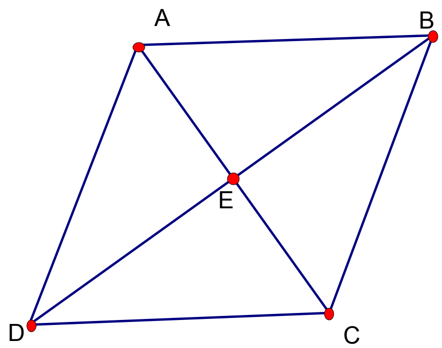
8. Consider the triangle defined by $P = (1, 3)$, $Q = (2, 5)$, and $R = (6, 5)$. The *transformation* defined by $T(x, y) = (x + 2, y - 1)$ is mathematical notation for translating a point by the vector [2, -1]. The point P(1, 3) becomes $T(1, 3) = (1 + 2, 3 - 1) = P'(3, 2)$. Find Q' and R'. Graph both the original triangle and its *image*.

9. (Continuation) The transformation $T(x, y) = (y + 2, x - 2)$ is a reflection. Verify this by calculating the effect of T on triangle PQR. Sketch triangle PQR, find coordinates for the *image points P'*, *Q'*, and *R'*, and sketch the *image triangle $P'Q'R'$*. Then identify the mirror line and add it to your sketch. Notice that triangle PQR is labeled in a clockwise sense; what about the labels on triangle $P''Q''R''$?

10. True or false? $\sqrt{4x} + \sqrt{9x} = \sqrt{13x}$

Fill in The Blanks:

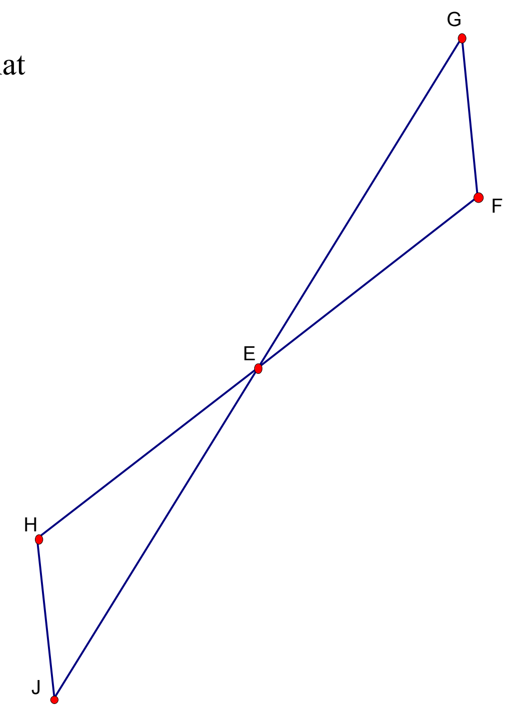
1. Prove that in a rhombus, the diagonals create four congruent triangles.



$AB \cong BC \cong CD \cong DA$ because
____.
Since B is equidistant from A and C, point B
____.
Since B lies on the perpendicular bisector of AC, segment________ ≅ segment ________.
Similarly, since C lies on the perpendicular bisector of DB, segment________ ≅ segment ________.
So by ________ we can say that
____
_____________________________________.

Therefore, four congruent triangles are formed.

2. Given the following picture, and that HF and JG bisect each other at point E, prove $\angle H \cong \angle F$.



Since HF and JG bisect each other at point E, we can say that

________________ ≅ __________________ and

________________ ≅ __________________.

We can also say that the pair of angles
∠ ______________ and ∠ _________________

are congruent to each other because the are vertical angles.

Therefore, by _______________
we can say that the triangles _____________ and _____________ are congruent.

So by ____________________, $\angle H \cong \angle F$.

GSP Lab #6

Medians

A. Open your sketch called My Special Segments. With your current triangle ABC highlight the perpendicular lines and choose *hide perpendicular lines* from the DISPLAY menu. You should still have the intact triangle and the point R on your sketch.
B. Highlight the three segments that are the sides of triangle ABC, and select *midpoint* from the CONSTRUCT menu.
C. Construct the medians of this triangle.
D. Construct the intersection of the medians and label this point P. This is called the **centroid** of the triangle.
E. Save this sketch.

Answer the following questions:

1. What can be said about the three medians of a triangle?
2. Do the properties that you observed for the orthocenter hold true for this point? Test your conjecture by making the triangle right, obtuse and acute.
3. This point, the centroid, has another special property:
    a. Highlight, in this order, vertex A, the midpoint of BC and the centroid.
    b. With all three highlighted, choose *Ratio* from the MEASURE menu.
    c. Do the same for the other two vertices.
4. What do these ratios tell you about the segments that are on the median?

1. Find the point on the line $3x + 2y = 16$ that is closest to the origin. Find the vector from the origin to the line.

2. In quadrilateral *ABCD*, it is given that $AB = CD$ and $BC = DA$. Prove that angles *ACD* and *CAB* are the same size. Note: If a polygon has more than three vertices, the *labeling convention* is to place the letters around the polygon in the order that they are listed. Thus, *AC* should be one of the diagonals of *ABCD*.

3. Find the area of the triangle defined by $E(-2, 8)$, $W(10, 2)$, and $S(-2, -4)$.

4. A segment from one of the vertices of a triangle to the midpoint of the opposite side is called a *median*. Consider the triangle defined by $A = (-2, 0)$, $B = (6, 0)$, and $C = (4, 6)$.
   (a) Find an equation for the line that contains the median drawn from *A* to *BC*.
   (b) Find an equation for the line that contains the median drawn from *B* to *AC*.
   (c) Find coordinates for *G*, the intersection of the medians from *A* and *B*.
   (d) Let *M* be the midpoint of *AB*. Determine whether or not *M*, *G*, and *C* are collinear.

1. The line $3x + 2y = 16$ is the perpendicular bisector of the segment AB. Find coordinates of point B, given that **(a)** $A = (-1, 3)$; **(b)** $A = (0, 3)$.

2. (Continuation) Point B is called the *reflection of A across the line* $3x + 2y = 16$; sometimes B is simply called the *image* of A. Explain this terminology. Using the same line, find another point C and its image C'. Explain your method for finding your pair of points.

3. A rhombus has 25-cm sides, and one diagonal is 14 cm long. How long is the other diagonal?

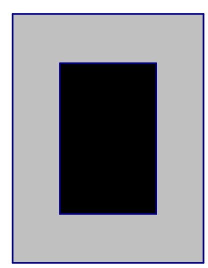
4. Let $A = (0, 0)$ and $B = (12, 5)$, and let C be the point on segment AB that is 8 units from A. Find coordinates for C.

5. Prove that one of the diagonals of a kite bisects two of the angles of the kite. Can you conclude the same thing about the other diagonal?

6. Let $A = (1, 4)$, $B = (8, 0)$, and $C = (7, 8)$. Find the area of triangle ABC.

7. Sketch triangle PQR, where $P = (1, 1)$, $Q = (1, 2)$, and $R = (3, 1)$. For each of the following, apply the given transformation T to the vertices of triangle PQR, sketch the image triangle $P'Q'R'$. It is advisable to sketch the respective images in different colors from the original triangle PQR. Then decide which of the terms *reflection*, *rotation*, *translation*, or *glide-reflection* accurately describes the action of T. Provide appropriate detail to justify your choices.
   (a) $T(x, y) = (x + 3,\ y - 2)$ **(b)** $T(x, y) = (y,\ x)$
   (c) $T(x, y) = (-x + 2, -y + 4)$ **(d)** $T(x, y) = (x + 3, -y)$

8. If the diagonals of a quadrilateral bisect each other, then any two nonadjacent sides are congruent. Prove that this is so. It is sufficient to show this for one pair of nonadjacent sides. For what kind of quadrilateral is this always true?



9. Robin is mowing a rectangular field that measures 24 yards by 32 yards, by pushing the mower around and around the outside of the plot. This creates a widening border that surrounds the un-mowed grass in the center. During a brief rest, Robin wonders whether the job is half done yet. How wide is the uniform mowed border when Robin *is* half done?

10. Triangle ABC is isosceles, with $AB = BC$, and angle $\boldsymbol{BAC}$ is 56 degrees. Find the remaining two angles of this triangle.

11. Terry walked one mile due north, two miles due east, then three miles due north again, and then once more east for 4 miles. How far is Terry from her starting point?

12. Triangle ABC is isosceles, with $AB = BC$, and angle $\boldsymbol{ABC}$ is 56 degrees. Find the remaining two angles of this triangle.