1. Find the area of the triangle whose vertices are $A = (-2, 3)$, $B = (6, 7)$, and $C = (0, 6)$.

2. Let $A = (-4, 0)$, $B = (0, 6)$, and $C = (6, 0)$.
   (a) Find equations for the three lines that contain the altitudes of triangle *ABC*.
   (b) Show that the three altitudes are *concurrent*, by finding coordinates for their common point. The point of concurrence is called the *orthocenter* of triangle *ABC*.

3. Pat and Chris were out in their rowboat one day, and Chris spied a water lily. Knowing that Pat liked a mathematical challenge, Chris announced that, with the help of the plant, it was possible to calculate the depth of the water under the boat. When pulled taut, the top of the plant was originally 10 inches above the water surface. While Pat held the top of the plant, which remained rooted to the lake bottom, Chris gently rowed the boat five feet. This forced Pat's hand to the water surface. Use this information to calculate the depth of the water.

4. Prove that if triangle *ABC* is isosceles, with $AB = AC$, then the medians drawn from vertices *B* and *C* must have the same length.

5. Find k so that the vectors [4,-3] and [k, -6]
   (a) point in the same direction; **(b)** are perpendicular.

6. Let $A = (-4, 0)$, $B = (0, 6)$, and $C = (6, 0)$.
   (a) Find equations for the three medians of triangle *ABC*.
   (b) Show that the three medians are concurrent, by finding coordinates for their common point. The point of concurrence is called the *centroid* of triangle *ABC*.

7. Given points $A = (0, 0)$ and $B = (-2, 7)$, find coordinates for *C* and *D* so that *ABCD* is a square.

8. Let $A = (0, 12)$ and $B = (25, 12)$. If possible, find coordinates for a point *P* on the x-axis that makes angle *APB* a right angle

9. The lines $3x + 4y = 12$ and $3x + 4y = 72$ are parallel. Explain why, and then find the distance that separates these lines. You will have to decide what "distance" means in this context.

10. Give an example of an equiangular polygon that is not equilateral.

11. On a separate sheet of paper, draw parallelogram *PQRS* with vertices at $P(0, 0)$, $Q(1, 3)$, $R(6, 2)$, and $S(5, -1)$. Cut out your parallelogram and dissect it to form a rectangle. What can you conclude about the area of a parallelogram?

12. Prove that a diagonal of a square divides it into two congruent triangles.

13. Find the area of the parallelogram whose vertices are (0, 0), (7, 2), (8, 5), and (1, 3).

1. Given the points $A = (0, 0)$, $B = (7, 1)$, and $D = (3, 4)$, find coordinates for the point C that makes quadrilateral $ABCD$ a parallelogram. What if the question had requested $ABDC$ instead?

2. Find a vector that is perpendicular to the line $3x - 4y = 6$.

3. Let $P = (-1, 3)$. Find the point Q for which the line $2x + y = 5$ serves as the perpendicular bisector of segment PQ.

GSP Lab #7

Perpendicular Bisectors

A. Open your sketch titled My Special Segments. Select all three medians (but not point P) and choose DISPLAY→Hide lines. Your triangle should still be intact with the two points R and P still in view. The midpoints of the sides should still be in view as well.
B. In turn, select each midpoint and its corresponding side and choose perpendicular line from the CONSTRUCT menu. You should have the three perpendicular bisectors of each side of the triangle.
C. Using the selection tool, click on the intersection point of all of the perpendicular bisectors. Let's call this point Q. This point is called the **circumcenter**.

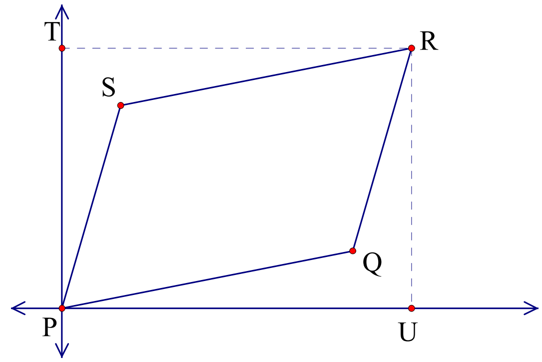
Answer the following questions:

1. Move your triangle around observe what happens to the circumcenter. What happens to this point when the triangle is right?
2. What happens to this point when the triangle is obtuse?
3. What happens to this point when the triangle is acute?
4. Why might the circumcenter and the orthocenter behave in the same ways with regards to the position of the triangle?
5. The circumcenter also has another interesting property. Recall the property of perpendicular bisectors discussed in class. The intersection of the perpendicular bisectors then has that property for both segments. So what do you think is true of the circumcenter?
6. Check your conjecture by selecting the circle tool. With the cursor click on the circumcenter and drag the mouse to one of the vertices of the triangle (it doesn't matter which one – why not?). Describe the circle in relation to the triangle.

4. Find points on the line $3x + 5y = 15$ that are equidistant from the coordinate axes.

5. Plot all points that are 3 units from the x-axis. Describe the configuration. Then, plot all the points 3 units from $(5, 4)$ and describe their configuration.

6. In triangle ABC, it is given that $CA = CB$. Points P and Q are marked on segments CA and CB, respectively, so that angles CBP and CAQ are the same size. Prove that $CP = CQ$.



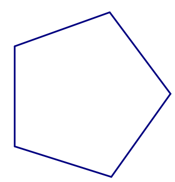
1. The figure at right shows a parallelogram *PQRS*, three of whose vertices are *P* = (0, 0), *Q* = (*a, b*), and *S* = (*c, d*). You can also see that *TRUP* is a rectangle. All of your expressions should be in terms of *a, b, c,* and *d*.
   (a) Find the coordinates of *R*.
   (b) Write an expression for the area of the rectangle *TRUP*.
   (c) Find expressions for the areas of triangles *TSR, TSP, PQU,* and *RQU.*
   (d) Find and expression for the area of *PQRS*, and simplify your formula.
   (e) There are two limitations on this formula. What do you think they are?

2. Let *A* = (3, 4), *B* = (0,-5), and *C* = (4,-3). Find equations for the perpendicular bisectors of segments *AB* and *BC*, and coordinates for their common point *K*. Calculate lengths *KA*, *KB*, and *KC*. Why is *K* also on the perpendicular bisector of segment *CA*?

3. (Continuation) A *circle* centered at *K* can be drawn so that it goes through all three vertices of triangle *ABC*. Explain. This is why *K* is called the *circumcenter* of the triangle. In general, how do you locate the circumcenter of a triangle?

4. *Some Terminology:* Draw a parallelogram whose *adjacent* edges are determined by vectors [2, 5] and [7, -1], placed so that they have a common initial point. This is called placing vectors *tail-to-tail*. Find the area of the parallelogram.

5. A polygon that is both equilateral and equiangular is called *regular*. Prove that all diagonals of a regular *pentagon* (five sides) have the same length.



6. Find coordinates for the point equidistant from (-1, 5), (8, 2), and (6, -2).

7. Solve for x: $\sqrt{x+1} = 7$

8. Write an equation that says that the distance from point (*x*, *y*) to (3, 5) is equal to the distance from the point (*x*, *y*) to (7, -1). There is no need to simplify your formula.

9. Find the area of the parallelogram whose vertices are (2, 5), (7, 6), (10, 10), and (5, 9).

10. Let *E* = (2, 7) and *F* = (10, 1). There are two points on line *EF* that are 3 units from *E*. Use vectors to find coordinates for both of them.

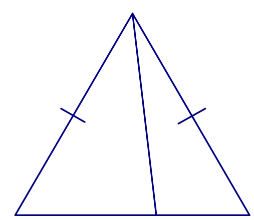
11. Let *A* = (1, 3), *B* = (7, 5), and *C* = (5, 9). Answer the item below that is determined by *the first letter of your last name*. Find coordinates for the requested point.
    (a-e) Show that the three medians of triangle *ABC* are concurrent at a point *G*.
    (f-m) Show that the three altitudes of triangle *ABC* are concurrent at a point *H*.
    (n-z) Show that the perpendicular bisectors of the sides of triangle *ABC* are concurrent at a point *K*. What special property does *K* have?

GSP Lab #8

A. Open your sketch titled My Special Segments and choose GRAPH→Define Coordinate System. A coordinate plane should appear behind your triangle and all points constructed. This will allow you to have some calculations at your disposal.
B. With the selection tool, select the orthocenter and the centroid and choose CONSTRUCT→segment. With the segment highlighted, select MEASURE→ slope.
C. Now select the orthocenter and the circumcenter and choose CONSTRUCT→segment and also measure its slope.
D. Now select a segment from the circumcenter to the centroid. Measure its slope as well.
E. Select one vertex of the triangle ABC and move it around on your sketch. What do you conjecture about these three points?

1. Simplify equation $\sqrt{(x-3)^2+(y-5)^2}=\sqrt{(x-7)^2+(y+1)^2}$. Interpret your result.

2. Use the diagram at right to help you explain why SSA evidence is not by itself sufficient to justify the congruence of triangles. The tick marks designate segments that have the same length.



3. You have recently seen that there is no generally reliable SSA criterion for congruence. If the angle part of such a correspondence is a *right* angle, however, the criterion *is* reliable. Justify this so-called *hypotenuse-leg* criterion (which is abbreviated HL).

4. Find an equation for the line through point (7, 9) that is perpendicular to vector [5, -2].

5. Describe a transformation that carries the triangle with vertices (1, 2), (6, 7), and (10, 5) onto the triangle with vertices (0, 0), (7, -1), and (9, 3). Where does your transformation send (7, 4)?

6. A triangle that has a 13-inch side, a 14-inch side, and a 15-inch side has an area of 84 square inches. Accepting this fact, find the lengths of all three altitudes of this triangle.

7. Find the area of a triangle having sides 10, 10, and 5.

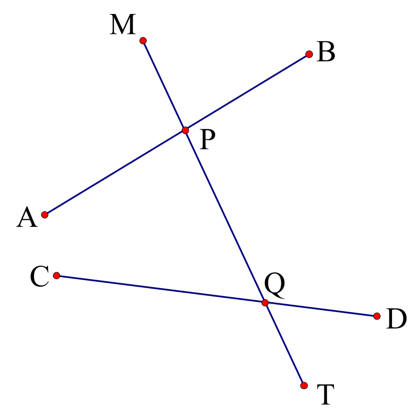
8. Find the lengths of *all* the altitudes of the triangle whose vertices are (0, 0), (3, 0), and (1, 4).

9. Let $P = (2, 7)$, $B = (6, 11)$, and $M = (5, 2)$. Find a point D that makes $\overrightarrow{PB}=\overrightarrow{DM}$. What can you say about quadrilateral *PBMD*?

10. Given that (-1, 4) is the reflected image of (5, 2), find an equation for the line of reflection.

1. Point (0, 1) is reflected across the line $2x + 3y = 6$. Find coordinates for its image.

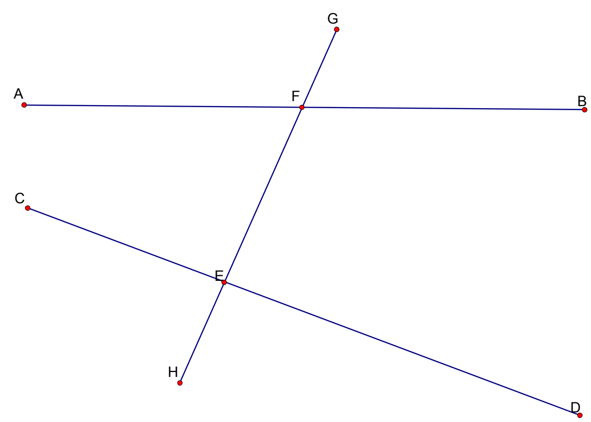
2. The diagram at right shows lines *APB* and *CQD* intersected by line *MPQT*, which is called a *transversal*. There are two groups of angles: one group of four angles with vertex at *P*, and another group with vertex at *Q*. There is special terminology to describe pairs of angles –one from each group. If the angles are on different sides of the transversal, they are called *alternate*, for example, angles *APM* and *PQD*. Angle *BPQ* is an *interior* angle because it is between the lines *AB* and *CD*, and angle *CQT* is *exterior*. Thus, angles *APQ* and *PQD* are called *alternate interior*, while angles *MPB* and *CQT* are called *alternate exterior*. On the other hand, the pair of angles *MPB* and *PQD* – which are non-alternate angles, one interior, and the other exterior – is called *corresponding*. Refer to the diagram and name
(a) the other pair of alternate interior angles;
(b) the other pair of alternate exterior angles;
(c) the angles that correspond to *CQT* and to *TQD*.



GSP Lab #9

In this lab, we will discover properties of angles formed by two lines (or segments) cut by a transversal.

A. Draw two segments, not necessarily parallel, and a transversal, as in the diagram. Construct the intersection points and label as shown in the diagram
B. Measure all eight of the angles using the MEASURE→Angle menu. Recall that in order to measure an angle, you need to select the three points that define that angle. The vertex of the angle must be the second point you highlight.
C. Move all angle measurements so that the values are close to the actual angle that you measured by dragging the measurement.
D. Using the arrow (selection tool), drag the endpoints of one of the segments (not the transversal). All angle measure should remain in view, but the measure of the angles should change.
E. Drag one of the segments so that they appear parallel to each other, but still crossed by the transversal.



GSP Lab #9 continues on the next page

F. Fill in the chart below with the values you observe for two such pairs (of each type) and the possible relationship.

Angle Type	First Pair measure	Second Pair measure	Relationship?
Corresponding			
Alternate Interior			
Same Side Interior			

G. For each type of angles, make a conjecture about the relationship of the lines. What is the requirement for your conjectures to be true?

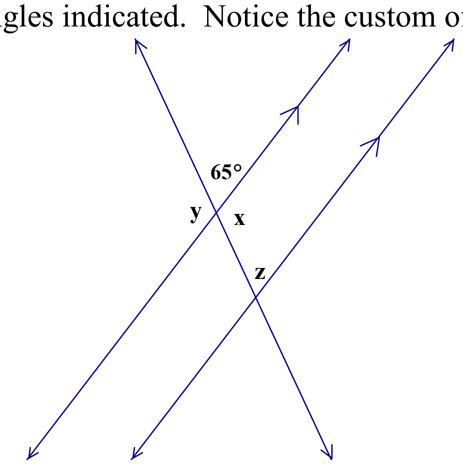
H. Are the converses of these statements true?

1. Find the area of a triangle formed by placing the vectors [3, 6] and [7, 1] tail-to-tail.

2. (Continuation) Describe your triangle using a different pair of vectors.

3. (Continuation) Find the length of the longest altitude of your triangle.

4. For the diagram at the right, find the measure of the angles indicated. Notice the custom of marking arrows on lines to indicate that they are known to be parallel.



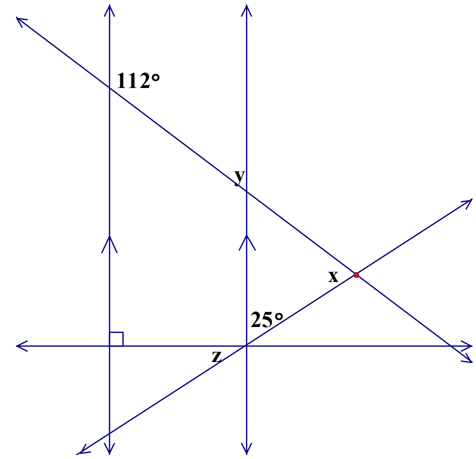
5. Given two parallel lines cut by a transversal, prove that a pair of alternate interior angles are congruent.

6. The diagonals of quadrilateral *ABCD* intersect perpendicularly at *O*. What can be said about quadrilateral *ABCD*?

7. What do you call **(a)** an *equiangular quadrilateral*? **(b)** an *equilateral quadrilateral*? **(c)** a *regular quadrilateral?*

8. In quadrilateral *ABCD*, it is given that $\overrightarrow{AB} = \overrightarrow{DC}$. What kind of a quadrilateral is *ABCD*? What can be said about the vectors $\overrightarrow{AD}$ and $\overrightarrow{BC}$?

9. Asked to reflect the point $P = (4, 0)$ across the mirror line $y = 2x$, Aubrey reasoned this way: First mark the point $A = (1, 2)$ on the line, then use the vector [-3, 2] from P to A to reach from A to $P' = (-2, 4)$, which is the requested image. What did Aubrey do wrong? Explain.



1. For the diagram at the right, find the measure of the angles indicated.

2. Given isosceles triangle *ABC* where *AB* = *BC* =10 and the altitude from *B* has length 4. Find the length of the base.

3. You probably know that the sum of the angles of a triangle is a straight angle. One way to confirm this is to draw a line through one of the vertices, parallel to the opposite side. This creates some alternate interior angles. Finish the demonstration.

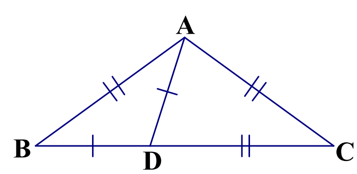
4. If it is known that one pair of alternate interior angles is equal, what can be said about **(a)** the other pair of alternate interior angles? **(b)** either pair of alternate exterior angles? **(c)** any pair of corresponding angles? **(d)** either pair of non-alternate interior angles?

5. Suppose that *ABCD* is a square and that *CDP* is an equilateral triangle, with *P outside* the square. What is the size of angle *PAD*?

6. Triangle *ABC* is isosceles with *AB* congruent to *AC*. *Extend* segment *BA* to a point *T* (in other words, *A* should be between *B* and *T*). Prove that angle *TAC* must be twice the size of angle *ABC*. Angle *TAC* is called one of the *exterior angles* of triangle *ABC*.

7. Recall that a quadrilateral that has two pairs of parallel opposite sides is called a *parallelogram*. What can be said about the angles of such a figure?

8. If *ABC* is any triangle, and *TAC* is one of its exterior angles, then what can be said about the size of angle *TAC*, in relation to the other angles of the figure?

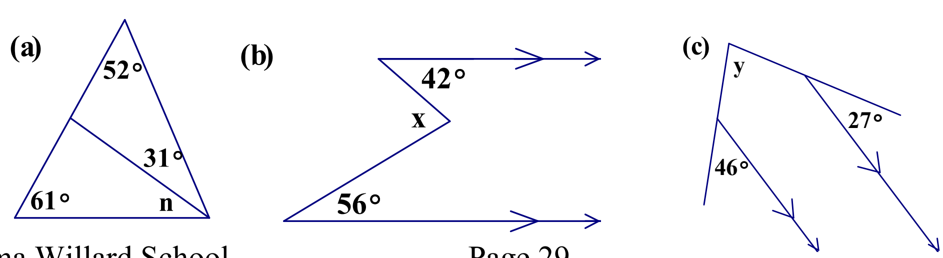


9. In the figure at right, it is given that *BDC* is straight, *BD* = *DA*, and *AB* = *AC* = *DC*. Find the size of angle *C*.

10. Given triangle *ABC*, with *AB* = *AC*, extend segment *AB* to a point *P* so that *B* is between *A* and *P* and *BP* = *BC*. In the resulting triangle *APC* show that angle *ACP* is exactly three times the size of angle *APC*. (By the way, notice that extending segment *AB* does *not* mean the same thing as extending segment *BA*.)

11. Given an arbitrary triangle, what can you say about the *sum* of the three exterior angles, one for each vertex of the triangle?

12. In the diagrams below, the goal is to find the sizes of the angles marked with letters, using the given numerical information.



1. Prove that the sum of the angles of any quadrilateral is 360 degrees.

2. Write the Pythagorean Theorem in if…then form. State the converse of the Pythagorean Theorem.

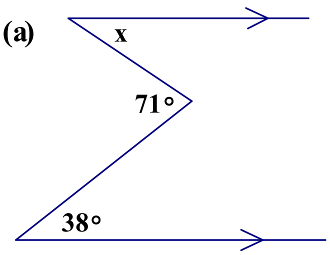
3. Fill in the following table

Number of sides of polygon	3	4	5	6	7	8	…	n
Number of non-overlapping triangles	1	2					…	
Total sum of the angles	180°	360°					…	
One Angle on a regular n-sided polygon							…	

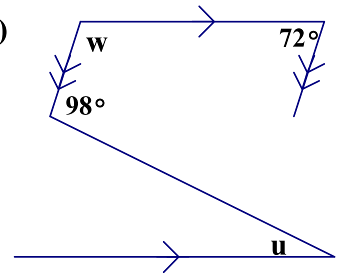
4. Given parallelogram $PQRS$, let T be the intersection of the bisectors of angles P and Q. Without knowing the sizes of the angles of $PQRS$, calculate the size of angle PTQ. Recall that the diagonals of a parallelogram are not necessarily the angle bisectors.

5. In the figures below, find the sizes of the angles indicated by letters:

(a)



(b)



6. Mark the point P inside square $ABCD$ that makes triangle CDP equilateral. Calculate the size of angle PAD.

7. The *converse* of a statement of the form "If A then B" is the statement "If B then A." Write the converse of the statement "If it is Tuesday, we have morning reports after first period"

8. (Continuation) "If point P is equidistant from the coordinate axes, then point P is on the line $y = x$".
   (a) Give an example of a true statement whose converse is false.
   (b) Give an example of a true statement whose converse is also true.

9. If a quadrilateral is a parallelogram, then both pairs of opposite angles are congruent. What is the converse of this statement? Is the converse true?

10. Find the measure of an interior angle of a regular decagon.

11. In regular pentagon $ABCDE$, draw diagonal AC. What are the sizes of the angles of triangle ABC? Prove that segments AC and DE are parallel.

1. Given square *ABCD*, let *P* and *Q* be the points outside the square that make triangles *CDP* and *BCQ* equilateral. Prove that triangle *APQ* is also equilateral.

2. The sides of an equilateral triangle are 12 cm long. How long is an altitude of this triangle? What are the angles of a right triangle created by drawing an altitude? How does the short side of this right triangle compare with the other two sides?

3. If a quadrilateral is a parallelogram, then both pairs of opposite sides are congruent. Explain. What is the converse of this statement? Is it true?

4. If the diagonals of a quadrilateral bisect each other then the figure is a parallelogram. Prove that this is so. What about the converse statement?

5. In triangle *ABC*, it is given that angle *A* is 59 degrees and angle *B* is 53 degrees. The altitude from *B* to line *AC* is extended until it intersects the line through *A* that is parallel to segment *BC*; they meet at *K*. Calculate the size of angle *AKB*.

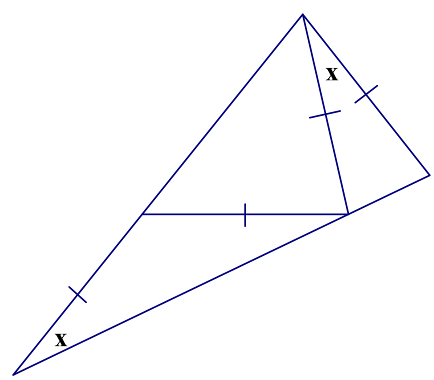
6. Given square *ABCD*, let *P* and *Q* be the points outside the square that make triangles *CDP* and *BCQ* equilateral. Segments *AQ* and *BP* intersect at *T*. Find angle *ATP*.

7. Using GSP, draw an acute-angled, non-equilateral triangle *ABC* and construct the circumcenter, *K*. Once you have constructed your circumcenter, hide your lines.
(a) Measure $\angle A$ and $\angle BKC$, $\angle B$ and $\angle CKA$, and $\angle C$ and $\angle AKB$. What do you conjecture is the relationship between these angles?
(b) Select side AB and construct its median. Measure the length of side AB and its median. Move the vertices of *ABC* so that the median is half the length of AB. What is true about this triangle?
(c) Is it true that the midpoint of the hypotenuse of a right triangle is equidistant to all three vertices? If so, why? If not, give a counterexample.

8. Tate walks along the boundary of a four-sided plot of land, writing down the number of degrees turned at each corner. What is the sum of these four numbers?

9. How can one tell whether a given quadrilateral is a parallelogram? In other words, how much evidence is needed to be sure of such a conclusion?

10. In the figure at right, there are two *x*-degree angles, and four of the segments are congruent as marked. Find *x*.



1. Jackie walks along the boundary of a five-sided plot of land, writing down the number of degrees turned at each corner. What is the sum of these five numbers?

2. Marty walks along the boundary of a seventy-sided plot of land, writing down the number of degrees turned at each corner. What is the sum of these seventy numbers?

3. The preceding two questions illustrate the *Sentry Theorem*. What does this theorem say, and why has it been given this name?

4. A rectangle with area 540 has one side of length 15. Find the length of the other side and the diagonals.

5. Can two of the angle bisectors of a triangle intersect perpendicularly? Explain.

6. A right triangle has a 24-cm perimeter and its hypotenuse is twice as long as its shorter leg. To the nearest tenth of a cm, find the lengths of all three sides of this triangle.

7. Let $A = (0, 0)$, $B = (8, 0)$, and $C = (x, x)$. Find x, given that **(a)** $BC = 4\sqrt{2}$; **(b)** $BC = 7$.

8. Let $A = (1, 1)$, $B = (3, 5)$, and $C = (7, 2)$. Explain how to cover the whole plane with non-overlapping triangles, each of which is congruent to triangle ABC.

9. (Continuation) In the pattern of lines produced by your *tessellation* you should see triangles of many different sizes. What can you say about their sizes and shapes?

10. The *midsegment* of a triangle is a segment that connects the midpoints of two sides of the triangle. Given a triangle with coordinates $A(1, 7)$, $B(5, 3)$ and $C(-1, 1)$ find the segment that connects the midpoints of sides AB and AC, label the midpoints M and N, respectively.
    (a) Find the length of the midsegment MN and compare it to the length of BC
    (b) What can be said about the lines containing segments BC and MN?

11. Suppose that quadrilateral $ABCD$ has the property that AB and CD are congruent and parallel. Is this enough information to prove that $ABCD$ is a parallelogram? Explain.

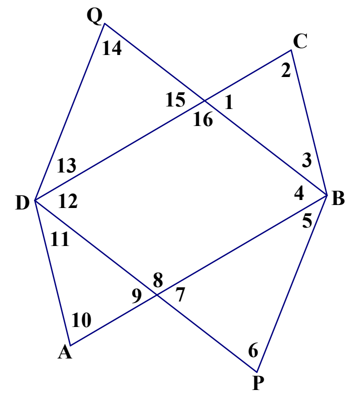
12. Given rectangle $ABCD$, let P be the point outside $ABCD$ that makes triangle CDP equilateral, and let Q be the point outside $ABCD$ that makes triangle BCQ equilateral. Prove that triangle APQ is also equilateral.

13. (Continuation) Would APQ be equilateral if $ABCD$ was a parallelogram?

14. A regular, n-sided polygon has 18-degree exterior angles. Find the integer n.

15. Draw a triangle ABC, and let AM and BN be two of its medians, which intersect at G. Extend AM to the point P that makes $GM = MP$. Prove that $PBGC$ is a parallelogram.

1. Triangle *EWS* has a perimeter of 26 and $SE = \frac{1}{2} WE$. The midsegment parallel to $WS = 4$. Find the lengths of the three sides of this triangle.



2. In the figure at right, it is given that *ABCD* and *PBQD* are parallelograms. Which of the numbered angles must be the same size as the angle numbered 1? Give a reason for each angle.

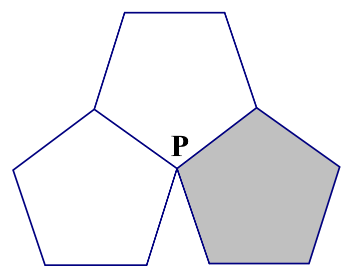
3. Triangle *PQR* has a right angle at *P*. Let *M* be the midpoint of *QR* and let *F* be the point where the altitude through *P* meets *QR*. Given that angle *FPM* is 18 degrees, find the sizes of angles *Q* and *R*.

4. Given that *ABCDEFG. . .* is a regular *n*-sided polygon, with angle *CAB* = 12 degrees, find *n*.

5. *Midsegment (Midline) Theorem:* State the properties of the segment that connects the midpoints of two sides of a triangle.

6. Draw triangle *ABC* so that angles *A* and *B* are both 42 degrees. Why should *AB* be longer than *BC*? Extend *CB* to *E*, so that *CB* = *BE*. Mark *D* on *AB* so that *DB* = *BC*, then draw the line *ED*, which intersects *AC* at *F*. Find the size of angle *CFD*.

7. The diagram at right shows three congruent regular pentagons that share a common vertex *P*. The three polygons do not quite surround *P*. Find the size of the uncovered acute angle at *P*.



8. (Continuation) If the shaded pentagon were removed, it could be replaced by a regular *n*-sided polygon that would exactly fill the remaining space. Find the value of *n* that makes the three polygons fit perfectly.

9. *The Centroid Theorem:* Using GSP, draw an acute, scalene triangle *ABC*, and two of its medians, *AM* and *BN*. Let *G* be the point where *AM* intersects *BN*.
   (a) Mark *GM* as a vector and translate *M* to a point *P* so that *GM* = *MP*. Draw a segment *MP* and change its color using DISPLAY→ Color.
   (b) Mark *GN* as a vector and translate *N* to a point *Q* so that *GN* = *NQ*. Draw a segment *NQ* and change its color using DISPLAY→ Color.
   (c) Draw in segment *PC* and make that the same color as segment *BG*. Draw in segment *QC* and make that the same color as *AG*. Why must these corresponding sides be parallel?
   (d) What type of quadrilateral is *PCQG*? Justify your answer (hint: look at the diagonals of this quadrilateral)
   (e) Find two segments in your sketch that must have the same length as *BG*. Measure them on your sketch to justify. What property of parallelograms also justifies this?
   (f) What must be the relationship of *BG* and *GN* considering the fact that *GN* = *NQ*?

1. A triangle is created by placing the vectors [7, 4] and [1, 3] tail-to-tail. State a vector that represents a midsegment of this triangle.

2. You are given a square *ABCD* and midpoints *M* and *N* are marked on *BC* and *CD*, respectively. Draw *AM* and *BN*, which meet at *Q*. Find the size of angle *AQB*.

3. Mark *Y* inside regular pentagon *PQRST*, so that *PQY* is equilateral. Is *RYT* straight? Explain.

4. Suppose that triangle *ABC* has a right angle at *B*, that *BF* is the altitude drawn from *B* to *AC*, and that *BN* is the median drawn from *B* to *AC*. Find angles *ANB* and *NBF*, given that angle *C* is 42 degrees.

5. The midpoints of the sides of a triangle are (3, -1), (4, 3), and (0, 5). Find coordinates for the vertices of the triangle.

6. We have discussed medians, perpendicular bisectors, altitudes, midsegments and angle bisectors of triangles.
   (a) Which of these *must* go through the vertices of the triangle?
   (b) Is it possible for a median to also be an altitude? Explain.
   (c) Is it possible for an altitude to also be a angle bisector? Explain.
   (d) Is it possible for a midsegment to be a median? Explain.
   (e) Is it possible for a perpendicular bisector to be an altitude?

7. The diagonals of a rhombus have lengths 18 and 24. How long are the sides of the rhombus?

8. A *trapezoid* is a quadrilateral with exactly one pair of parallel sides. If the non-parallel sides have the same length, the trapezoid is *isosceles*. Make a diagram of an isosceles trapezoid whose sides have lengths 7 in, 10 in, 19 in, and 10 in. Find the *altitude* of this trapezoid (the distance that separates the parallel sides), then find the enclosed area.

9. If a quadrilateral is a rectangle, then its diagonals have the same length. What is the converse of this true statement? Is the converse true? Explain.

10. The diagonals of a parallelogram always bisect each other. Is it possible for the diagonals of a trapezoid to bisect each other? Explain.

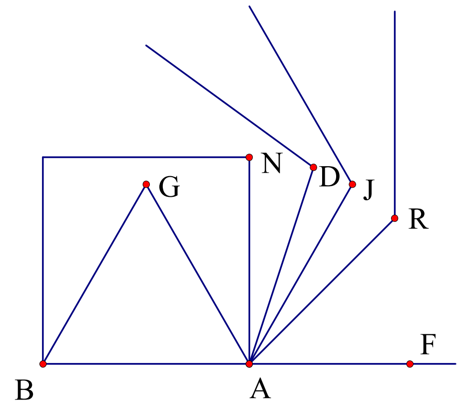
11. A trapezoid has a 60-degree angle and a 45-degree angle. What are the other angles?

12. A trapezoid has a 60-degree angle and a 120-degree angle. What are the other angles?

1. The sides of a triangle have lengths 9, 12, and 15. (This is a special triangle!)
   (a) Find the lengths of the medians of the triangle.
   (b) The medians intersect at the centroid of the triangle. How far is the centroid from each of the vertices of the triangle?

2. (Continuation) Apply the same questions to an equilateral triangle of side 6.

3. An *n*-sided polygon has the property that the sum of the measures of its exterior angles is equal to the sum of the measures of its interior angles. Find *n*.



4. In the diagram at right, *AGB* is an equilateral triangle, $\overline{AN}$ is the side of a square. $\overline{AD}$ is the side of a regular pentagon, $\overline{AJ}$ is the side of a regular hexagon, and $\overline{AR}$ is the side of a regular octagon. Find
   (a) $\angle GAF$ **(b)** $\angle NAR$ **(c)** $\angle JAF$ **(d)** $\angle GAJ$

5. Trapezoid *ABCD* has parallel sides *AB* and *CD*, a right angle at *D*, and the lengths *AB* = 15, *BC* = 10, and *CD* = 7. Find the length *DA*.

6. A line of positive slope is drawn so that it makes a 60-degree angle where it intersects the *x*-axis. What is the slope of this line?

7. What can be said about quadrilateral *ABCD* if it has supplementary adjacent angles?

8. If *MNPQRSTUV* is a regular polygon, then how large is each of its interior angles? If *MN* and *QP* are extended to meet at *A* then how large is angle *PAN*?

9. Is it possible for the sides of a triangle to be 23, 19, and 44? Explain.

10. Suppose that *ABCD* is a square with *AB* = 6. Let *N* be the midpoint of *CD* and *F* be the intersection of *AN* and *BD*. What is the length of *AF*? Hint: Look at triangle ADC.

11. An isosceles trapezoid must have two pairs of equal adjacent angles. State and prove the converse.

12. Draw the lines $y = 0$, $y = \frac{1}{2}x$, and $y = \frac{4}{3}x$. Use your protractor to measure the angles then make calculations to confirm what you observe.

13. The parallel sides of trapezoid *ABCD* are *AD* and *BC*. Given that sides *AB*, *BC*, and *CD* are each half as long as side *AD*, find the size of angle *D*.

14. Dana buys a piece of carpet that measures 20 square yards. Will Dana be able to completely cover a rectangular floor that measures 12 ft. 4 in. by 16 ft. 8 in.?

1. The altitudes of an equilateral triangle all have length 12 cm. How long are its sides?

2. Given a triangle, you have found the following result: *The point where two medians intersect (the centroid) is twice as far from one end of a median as it is from the other end of the same median*. Improve the statement of the preceding theorem so that the reader knows which end of the median is which. This theorem indirectly shows that the three medians of any triangle must be *concurrent*. Explain the reasoning.

3. Let *ABCD* be a parallelogram, with *M* the midpoint of *DA*, and diagonal *AC* of length 36. Let *G* be the intersection of *MB* and *AC* and draw in diagonal *DB*. What is the length of *AG*?

4. The diagonals of a square have length 10. How long are the sides of the square?

5. Triangle *PQR* is isosceles, with *PQ* = 13 = *PR* and *QR* = 10. Find the distance from *P* to the centroid of *PQR*. Find the distance from *Q* to the centroid of *PQR*.

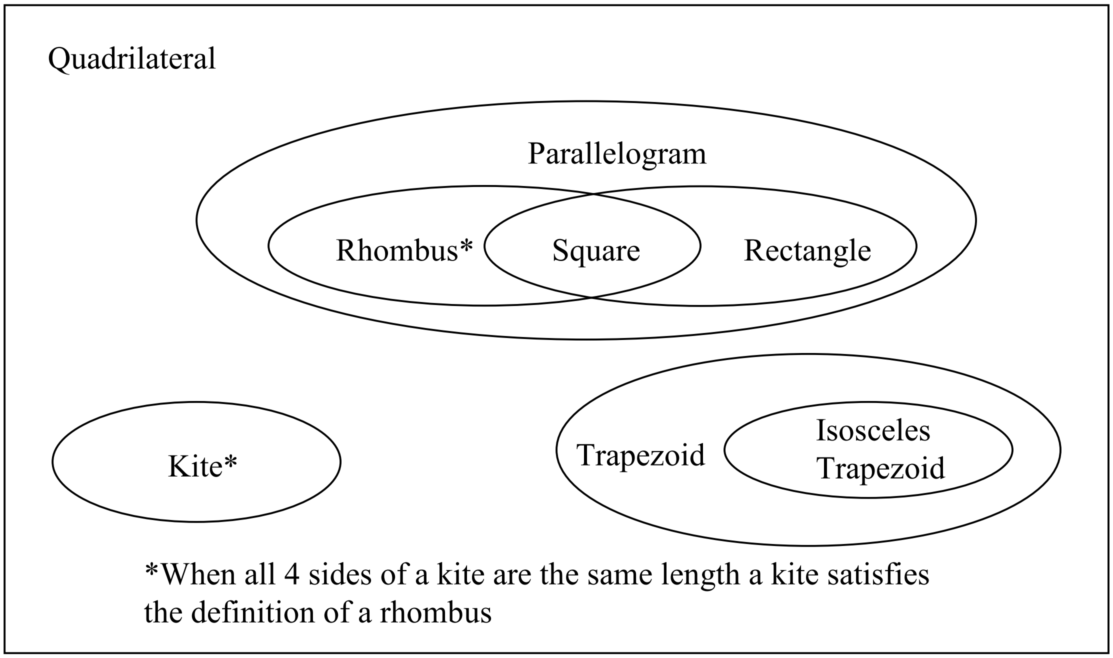
6. Using GSP, draw a long segment *AB* and two points, *C* and *E* below segment *AB*.
   (a) Highlight *C*, *E*, and the segment *AB* (not the endpoints) and choose CONSTRUCT→ parallel lines
   (b) Draw a transversal that cuts through the three parallel lines and construct the intersection points of the transversal and the three parallel lines. Label the point on the line through *C*, as *D*, and the point on the line through *E*, as *F*, and the intersection of *AB* and the transversal as *P*.
   (c) Select points *P* and *D* and choose MEASURE→distance, and then do the same with *D* and *F*. What happens to these distances when you drag the transversal horizontally thereby changing its slope?
   (d) Select *P*, *F* and *D* (in that order) and chose MEASURE→Ratio. What happens to the ratio of these distances when you drag the transversal horizontally in the sketch?
   (e) Draw another transversal, with a different slope than the first, and construct the intersection points with *AB*, *CD* and *EF*, called *R*, S, and *T* respectively.
   (f) Select *R*, *T* and *S* in that order and choose MEASURE→Ratio. What conclusions can you draw from this information?

7. In triangle *ABC*, let *M* be the midpoint of *AB* and *N* be the midpoint of *AC*. Suppose that you measure *MN* and find it to be 7.3 cm long. How long would *BC* be, if you measured it? If you were to measure angles *AMN* and *ABC*, what would you find?

8. In triangle *SUN*, let *P* be the midpoint of segment *SU* and let *Q* be the midpoint of segment *SN*. Draw the line through *P* parallel to segment *SN* and the line through *Q* parallel to segment *SU*; these lines intersect at *J*. What can you say about the location of point *J*?

9. A bell rope, passing through the ceiling above, just barely reaches the belfry floor. When one pulls the rope to the wall, keeping the rope taut, it reaches a point that is three inches above the floor. It is four feet from the wall to the rope when the rope is hanging freely. How high is the ceiling? It is advisable to make a clear diagram for this problem.

Justify the following Venn diagram and check the properties that each type of quadrilateral holds.

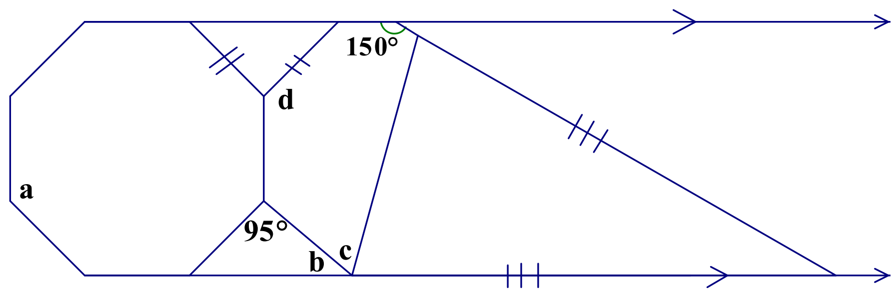


Property	Parallel-ogram	Rectangle	Rhombus	Square	Kite	Trapezoid	Isosceles Trapezoid
Opposite sides are parallel							
Opposite sides are congruent							
Exactly one pair of opposite sid es is congruent							
Opposite angles are congruent							
Exactly one pair of angles is congruent							
Consecutive angles are supplementary							
Base angles are congruent							
Diagonals bisect each other							
Diagonals are congruent							
Diagonals are perpendicular							
Diagonals bisect opposite angles							
Exactly one diagonal is the perpendicular bisector of the other							

1. A triangle with sides of 5, 12, and 13 must be a right triangle. Keeping the legs constant, how would the triangle change if the hypotenuse was lengthened to 15? 17? 19?

2. (Continuation) What can be said about a triangle if the sum of the squares of the two shorter sides is smaller than the square of the longest side?

3. In the diagram below the octagon is regular. Find the measures of the angles labeled a, b, c, and d.



4. What if the hypotenuse of a 5-12-13 triangle was shortened to 12? 7? 5?

5. (Continuation) What can be said about a triangle if the sum of the squares of the two shorter sides is larger than the square of the longest side?

6. Mark $A = (0, 0)$ and $B = (10, 0)$ on your graph paper, and use your protractor to draw the line of positive slope through A that makes a 25-degree angle with AB. Calculate (approximately) the slope of this line by making suitable measurements.

7. (Continuation) Turn on your calculator, press the MODE button, and select the *Degree* option for angles. Return to the home screen, and ENTER the expression TAN(25). You should see that the display agrees with your answer to the preceding item.

8. How does the value of TAN(angle) change as an angle increase 0 to 90 degrees?

9. A line drawn parallel to the side BC of triangle ABC intersects side AB at P and side AC at Q. The measurements $AP = 3.8$ in, $PB = 7.6$ in, and $AQ = 5.6$ in are made. If segment QC were now measured, how long would it be?

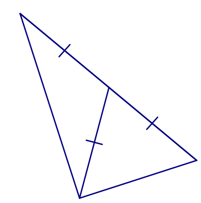
10. Standing 50 meters from the base of a fir tree, Rory measured an *angle of elevation* of 33° to the top of the tree with a protractor. The angle of elevation is the angle formed by the horizontal and the line of sight ray. How tall was the tree?

11. Given regular hexagon *BAGELS*, show that *SEA* is an equilateral triangle.

12. When the Sun has risen 32 degrees above the horizon, Sandy casts a shadow that is 9 feet 2 inches long. How tall is Sandy, to the nearest inch?

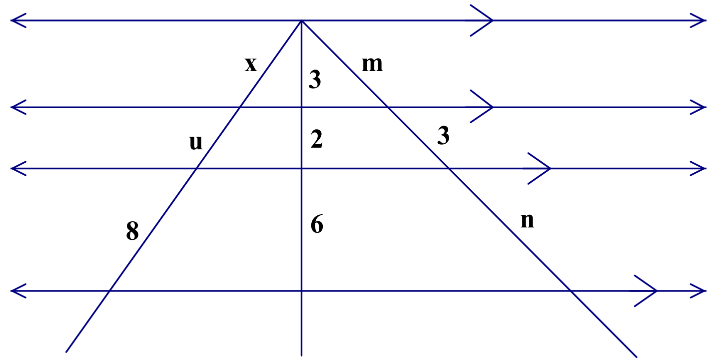
In the following list of true statements, find **(a)** the four pairs of statements whose converses are also in the list; **(b)** the statement that is a definition; **(c)** the statement whose converse is false; **(d)** the Sentry Theorem;**(e)** the Midsegment Theorem;**(f)** The Three Parallels Theorem; **(g)** The Centroid Theorem. Note: Not all statements are used.

1. If a quadrilateral has two pairs of parallel sides, then its diagonals bisect each other.
2. If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral must be a parallelogram.
3. If a quadrilateral is equilateral, then it is a rhombus.
4. If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



5. If a quadrilateral has two pairs of equal adjacent sides, then its diagonals are perpendicular.
6. If one of the medians of a triangle is half the length of the side to which it is drawn, then the triangle is a right triangle.
7. If a segment joins two of the midpoints of the sides of a triangle, then is parallel to the third side, and is half the length of the third side.
8. Both pairs of opposite sides of a parallelogram are congruent.
9. The sum of the exterior angles of any polygon – one at each vertex – is 360 degrees.
10. The median drawn to the hypotenuse of a right triangle is half the length of the hypotenuse.
11. If two lines are intersected by a transversal so that alternate interior angles are equal, then the lines must be parallel.
12. If the diagonals of a quadrilateral bisect each other, then the quadrilateral is in fact a parallelogram.
13. If two opposite sides of a quadrilateral are both parallel and equal in length, then the quadrilateral is a parallelogram.
14. If three parallel lines intercept equal segments on one transversal, then they intercept equal segments on every transversal.
15. Both pairs of opposite angles of a parallelogram are congruent.
16. The medians of any triangle are concurrent at a point that is two thirds of the way from any vertex to the midpoint of the opposite side.
17. An exterior angle of a triangle is the sum of the two nonadjacent interior angles.

1. The *Three Parallels* Theorem: If a transversal cuts three parallel lines in a given ratio, then any transversal cuts off segments of the same ratio. Use this to solve for x, u, m, and n in the following diagram.



2. Given $A = (0, 6)$, $B = (-8, 0)$, and $C = (8, 0)$, find coordinates for the circumcenter of triangle ABC.

3. Rearrange the letters of *doctrine* to spell a familiar mathematical word.

4. Standing on a cliff 380 meters above the sea, Pocahontas sees an approaching ship and measures its *angle of depression*, obtaining 9 degrees. How far from shore is the ship?

5. (Continuation) Now Pocahontas sights a second ship beyond the first. The angle of depression of the second ship is 5 degrees. How far apart are the ships?

6. What is the radius of the smallest circle that encloses an equilateral triangle with 12-inch sides? What is the radius of the largest circle that will fit inside the same triangle?

7. Let $A = (0, 0)$, $B = (4, 0)$, and $C = (4, 3)$. Measure angle CAB with your protractor. What is the slope of AC? Use your calculator to compare the tangent of the angle you measured with the slope. By trial-and-error, find an angle that is a better approximation of the measure of angle CAB.

8. (Continuation) On your calculator, ENTER the expression $\text{TAN}^{-1}(0.75)$. Compare this answer with the approximation you obtained for the measure of angle CAB. What does the TAN^{-1} button do? (TAN^{-1} is said as "inverse tangent.")

9. A five-foot Emma student casts an eight-foot shadow. How high is the Sun in the sky? This is another way of asking for the angle of elevation of the Sun.

10. An isosceles trapezoid has sides of lengths 9, 10, 21, and 10. Find the distance that separates the parallel sides then find the length of the diagonals. Finally, find the angles of the trapezoid, to the nearest tenth of a degree.

1. One day at the beach, Kelly flies a kite, whose string makes a 37-degree elevation angle with the ground. Kelly is 130 feet from the point directly below the kite. How high above the ground is the kite, to the nearest foot?

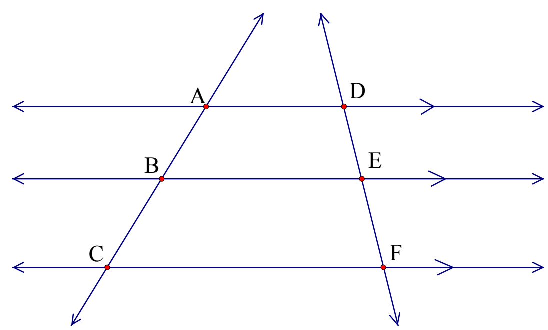
2. Hexagon *ABCDEF* is regular. Prove that segments *AE* and *ED* are perpendicular.

3. What angle does the line $y = \frac{2}{5}x$ make with the x-axis?

4. Suppose that *PQRS* is a rhombus, with *PQ* = 12 and a 60-degree angle at *Q*. How long are the diagonals *PR* and *QS*?

5. Given a rectangular card that is 5 inches long and 3 inches wide, what does it mean for another rectangular card to have the *same shape*? Describe a couple of examples.

6. *The Varignon quadrilateral*. A quadrilateral has diagonals of lengths 8 and 10. The midpoints of the sides of this figure are joined to form a new quadrilateral. What is the perimeter of the new quadrilateral? What is special about it?



7. Three parallel lines are cut by two transversals as shown in the figure at right. What do you think can be said about the segments AB, BC, DE and EF?

8. The diagonals of rhombus *ABCD* meet at *M*. Angle *DAB* measures 60 degrees. Let *P* be the midpoint of *AD* and let *G* be the intersection of *PC* and *MD*. Given that *AP* = 8, find *MD*, *MC*, *MG*, *CG*, and *GP*.

9. The hypotenuse of a right triangle is twice as long as one of the legs. How long is the other leg? What is the size of the smallest angle?

10. What are the angle sizes in a trapezoid whose sides have lengths 6, 20, 6, and 26?

11. Let *A* = (0, 0), *B* = (4, -3), *C* = (6, 3), *P* = (-2, 7), *Q* = (9, 5), and *R* = (7, 19). Using your protractor, measure the angles of triangles *ABC* and *PQR*. Calculate the lengths of the sides of these triangles. Find justification for any conclusions you make.

12. In triangle *ABC*, points *M* and *N* are marked on sides *AB* and *AC*, respectively, so that $AM : AB = 17:100 = AN : AC$. Why are segments *MN* and *BC* parallel?

13. Atiba wants to measure the width of the Hudson River. Standing under a tree *T* on the river bank, Atiba sights a rock at the nearest point *R* on the opposite bank. Then Atiba walks to a point *P* on the river bank that is 50.0 meters from *T*, and makes *RTP* a right angle. Atiba then measures *RPT* and obtains 76.8 degrees. How wide is the river?

14. The legs of an isosceles right triangle have a length of s. What is the length of the hypotenuse with respect to s?

1. A regular *n*-sided polygon has exterior angles of *m* degrees each. Express *m* in terms of *n*.

2. Out for a walk in Chicago, Morgan measured the angle of elevation to the distant Sears Tower, and got 3.6 degrees. After walking one km directly toward the building, Morgan found that the angle of elevation had increased to 4.2 degrees. Use this information to estimate the height of the Sears Tower and how far Morgan is from it after walking toward the building.

3. Rectangle *ABCD* has dimensions $AB = 5$ and $BC = 12$. Let *M* be the midpoint of *BC* and let *G* be the intersection of *AM* and diagonal *BD*. Find *BG* and *AG*.

4. Sketch a random quadrilateral *ABCD* with perpendicular diagonals *AC* and *BD* intersecting at point *E*. Let $BE = h_1$ and $DE = h_2$. By finding expressions for the area of triangles *ADC* and *ABC*, find a formula for the area of a quadrilateral that has perpendicular diagonals.

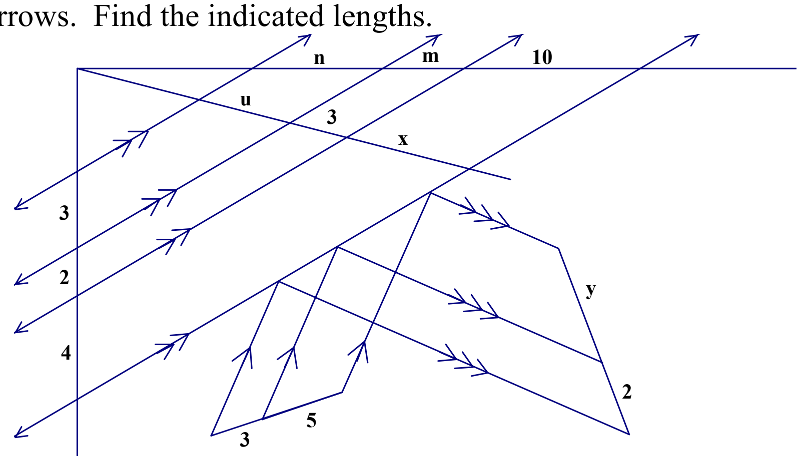
5. How tall is an isosceles triangle, given that its base is 30 cm long and that both of its base angles are 72 degrees?

6. A triangle has sides in the ratio $1:2:\sqrt{3}$. Draw a triangle with this scale. What can you say about this triangle?

7. In the Emma Willard class of 2010 there are 56 students, and the day**:** boarder ratio is $3:4$.
   (a) How many students in the class of 2010 are boarders?
   (b) How many day students would you expect to find in a freshman seminar class of fifteen students? Explain.

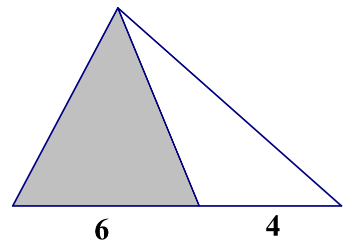
8. Find the equation of a line passing through the origin that makes an angle of 52 degrees with the *x*-axis.

9. In the figure below, find the lengths of the segments indicated by letters. Parallel lines are indicated by arrows. Find the indicated lengths.



1. *Special Right Triangles*. There are special right triangles with integer valued sides. These are called *Pythagorean Triples*. There are two other special right triangles commonly used in mathematics that do not have integer valued sides. One of these is a 45-45-90 triangle. What is the other one?

2. In the figure at right, the shaded triangle has area 15. Find the area of the unshaded triangle.



3. To the nearest tenth of a degree, how large are the congruent angles of an isosceles triangle that is exactly as tall as it is wide? (There is more than one interpretation).

4. Rectangle *ABCD* has *AB* = 16 and *BC* = 6. Let *M* be the midpoint of side *AD* and *N* be the midpoint of side *CD*. Segments *CM* and *AN* intersect at *G*. Find the length *AG*.

5. An estate of $362880 is to be divided among three heirs, Alden, Blair, and Cary. According to the will, Alden is to get two parts, Blair three parts, and Cary four parts. What does this mean in terms of dollars and cents?

6. What is the relationship between the length of the hypotenuse and the length of the legs in a 45-45-90 triangle?

7. The area of a parallelogram can be found by multiplying the distance between two parallel sides by the length of either of those sides. Explain why this formula works.

8. The perimeter of a square is 36, what is the length of a diagonal of the square? The area of a square is 36, what is the length of a diagonal of the square?

9. Given that *P* is three fifths of the way from *A* to *B*, and that *Q* is one third of the way from *P* to *B*, describe the location of *Q* in relation to *A* and *B*.

10. Apply the transformation $T(x, y) = (3x, 3y)$ to the triangle *PQR* whose vertices are $P = (3, -1)$, $Q = (1, 2)$, and $R = (4, 3)$. Compare the sides and angles of the image triangle *P'Q'R'* with the corresponding parts of *PQR*. This transformation is an example of a *dilation*.

11. Show that the altitude drawn to the hypotenuse of any right triangle divides the triangle into two triangles that have the same angles as the original.

12. State the relationship of the sides of a 30-60-90 triangle if the shortest side is *s*.

13. Suppose that the points *A*, *P*, *Q*, and *B* appear in this order on a line, such that $AP : AB = 3 : 5$ and $PQ : QB = 1 : 2$. Evaluate the ratios $AQ : AB$ and $AQ : QB$.

1. Compare the quadrilateral whose vertices are $A = (0, 0)$, $B = (6, 2)$, $C = (5, 5)$, $D = (-1, 3)$ with the quadrilateral whose vertices are $P = (9, 0)$, $Q = (9, 2)$, $R = (8, 2)$, and $S = (8, 0)$. Calculate lengths and angles, and look for patterns.

1. You have learned that the formula for the area of a triangle is $\frac{1}{2}bh$. Find a formula for the area of an equilateral triangle in terms of the side, *s*. Hint: Remember that an equilateral triangle is made up of two 30-60-90 triangles.

2. Draw a right triangle that has a 15-cm hypotenuse and a 27-degree angle. To the nearest tenth of a cm, measure the side opposite the 27-degree angle, and then express your answer as a percentage of the length of the hypotenuse. Compare your answer with the value obtained from your calculator when you enter SIN 27 in degree mode.

3. (Continuation) Repeat the process on a right triangle that has a 10-cm hypotenuse and a 65-degree angle. Try an example of your choosing. Write a summary of your findings.

4. The area of a trapezoid can be found by multiplying its altitude (the distance between the parallel sides) by the average of the bases. Explain why this formula works.

5. What is the length of an altitude of an equilateral triangle with perimeter 36?

6. To actually draw a right triangle that has a 1-degree angle and measure its sides accurately is difficult. To get the sine ratio for a 1-degree angle, however, there is an easy way – just use your calculator. Is the ratio a small or large number? How large can a sine ratio be?

7. If triangle *ABC* has a right angle at *C*, the ratio $AC : AB$ is called the *sine ratio of angle B*, or simply the *sine of B*, and is usually written *sin B*. What should the ratio $BC : AB$ be called? Without using your calculator, can you predict what the value of the sine ratio for a 30-degree angle is? How about the sine ratio for a 60-degree angle?

8. If two sides of a triangle are 5 and 10, what is the range of values for the third side?

GSP Lab #10

Using GSP, you will consider the transformation of a dilation.

A. Open a new GSP sketch and choose GRAPH→Plot Points and graph the points $C = (1, 4)$, $P = (5, 2)$ and $P' = (13, -2)$.
B. Select *C, P* and *P'* (in that order) and choose MEASURE→ratio, and the ratio of *CP'*/*CP* should appear. What is the ratio? This is called the *scale factor* of the dilation.
C. Now graph $Q = (3, 5)$. Select point *C* and choose TRANSFORM→Mark Center, to tell the program that you would like to use *C* as the center of a dilation.
D. Select point *Q* and choose TRANSFORM→Dilate. Name this image point *Q'*. In the pop-up window, you will tell the program the fixed ratio (i.e. scale factor) to use. The numerator of the fixed ratio is how many times larger you want the image to be. What would you enter for the fixed ratio if you wanted to use the same scale factor as *P*→*P'*?