GSP Lab #10, continued from previous page

E. Construct segments *PQ* and *P'Q'*. Select these segments in that order and choose MEASURE→ratio. What is the ratio of *PQ* to *P'Q'*? Did you expect this? Why?
F. What can be said about the points *C, P* and *P*'? What can be said about the points *C, Q* and *Q'*? What property does the center of dilation hold?
G. If *R'*=(-2,-2), what were the coordinates of *R*, its *pre-image*, using the same center of dilation? (hint: dilations can also make images closer to the center of dilation)

1. Let $C = (1, 4)$, $P = (5, 2)$, and $P' = (13, -2)$. There is a dilation that leaves *C* where it is and transforms *P* into *P'*. The point *C* is called the *dilation center*. Explain why the *magnitude* of this dilation is 3. Calculate *Q'*, given that $Q = (3, 5)$. Calculate *R*, given that $R' = (-6, 7)$.

2. What is the size of the acute angle formed by the *x*-axis and the line $3x + 2y = 12$?

3. One figure is *similar* to another figure if the points of the first figure can be matched with the points of the second figure in such a way that corresponding distances are proportional. In other words, there is a *ratio of similarity, k*, such that every distance on the second figure is *k* times the corresponding distances on the first figure.
   (a) Open a new sketch on GSP and choose GRAPH→plot points and plot the points $K = (1, -3)$, $L = (4, 1)$, $M = (2, 3)$, $P = (6, 5)$, $Q = (2, 5)$ and $R = (7, -2)$.
   (b) Is triangle *KLM* similar to triangle *PQR*? Justify with measurements from GSP.
   (c) Would it be correct to say that triangle *MKL* is similar to triangle *RQP*?

4. Write an equation using the distance formula that says that $P = (x, y)$ is 5 units from (0, 0). Plot several such points. What is the configuration of all such points called? How many are lattice points?

5. (Continuation) Explain how you could use the Pythagorean Theorem to obtain the same result.

6. Does a dilation transform any figure into a similar figure? If you know that two triangles are similar does that mean that they are dilations of one another?

7. What is the length of a side of an equilateral triangle whose altitude is 16?

8. When you take the sine of 30 degrees using your calculator you get 0.5. What do you think $\text{SIN}^{-1}(0.5)$ is? Use your calculator to test your conjecture. Find $\text{SIN}^{-1}(0.3)$ and $\text{SIN}^{-1}\left(\frac{3}{5}\right)$.
   What do these values represent?

9. When triangle *ABC* is similar to triangle *PQR*, with *A*, *B*, and *C* corresponding to *P*, *Q*, and *R*, respectively, it is customary to write $ABC \sim PQR$. Suppose *that* $AB = 4$, $BC = 5$, $CA = 6$, and $RP = 9$. Find *PQ* and *QR*.

1. To the nearest tenth of a degree, find the sizes of the acute angles in the right triangle whose hypotenuse is 2.5 times as long as its short leg.

2. A triangle has a 60-degree angle and a 45-degree angle and the side opposite the 45-degree angle is 240 mm long. To the nearest mm, how long is the side opposite the 60-degree angle?

3. Let $A = (0, 0)$, $B = (15, 0)$, $C = (5, 8)$, $D = (9, 0)$, and $P = (6, 6)$. Draw triangle ABC, segments CD, PA, and PB, and notice that P is on segment CD. There are now three pairs of triangles in the figure whose **areas** are in a 3: 2 ratio. Find them, and justify your choices.

4. If $74 is worth 54 Euros, what is the conversion factor of Dollars to Euros?

5. One triangle has sides that are 5 cm, 7 cm, and 8 cm long; the longest side of a similar triangle is 6 cm long. How long are the other two sides?

6. Judy is driving along a highway that is climbing a steady 9-degree slope. After driving for two miles along this road, how much altitude has Judy gained?

7. (Continuation) How far must Judy travel in order to gain a mile of altitude?

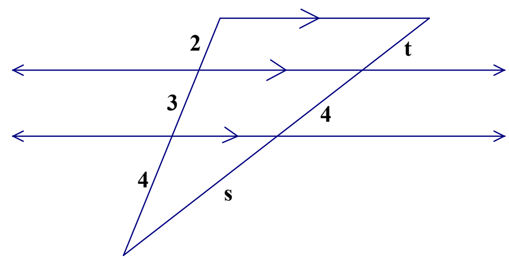
8. The floor plan of a house is drawn with a ratio of 1/8 inch = 1 foot. On the plan, the kitchen measures 2 in. by 2 1/4 in. What is the size of the kitchen?

9. If an altitude is also the side of a triangle, what do you know about the triangle?

10. Explain why corresponding angles of similar polygons are necessarily the same size.

11. (Continuation) If all the angles of a triangle are equal in size to the angles of another triangle, the triangles are similar. Justify this statement. Is this the converse of the preceding?

12. In the diagram at the right, find t and s.



13. Is it possible to draw a triangle with the given sides? If it is possible, state whether it is acute, right, or obtuse. If it is not possible, say no and sketch why.
(a) 9, 6, 5 **(b)** $3\sqrt{3}$, 9, $6\sqrt{3}$ **(c)** 8.6, 2.4, 6.2

14. *Discovery of π:* The Greek scholar Archimedes discovered a constant relationship between the circumference of a circle and its *diameter*. He called this constant π. Describe the circumference with respect to its diameter. With respect to its radius.

1. The area of an equilateral triangle with *m*-inch sides is 8 square inches. What is the area of a regular hexagon that has *m*-inch sides?

2. A parallelogram has 10-inch and 18-inch sides and an area of 144 square inches.
(a) How far apart are the 18-inch sides? **(b)** How far apart are the 10-inch sides?
(c) What are the angles of the parallelogram? **(d)** How long are the diagonals?

3. Write an equation that describes all the points on the circle whose *center* is at the origin and whose *radius* is **(a)** 13; **(b)** 6; **(c)** *r*.

GSP Lab #11

Dilation Exploration

A. Open a new GSP Sketch and construct an acute, scalene triangle *EWS*.
B. Construct a point somewhere else on the sketch (do not plot a point, you will need this point to be dynamic). Label this point *C*.
C. Double click on *C*, and you should see a short animation that tells you that the program is remember *C* as the center of dilation (shortcut for choosing TRANSFORM→mark center).
D. Select the entirety of triangle *EWS* and choose TRANSFORM→Dilate. In the pop-up window, leave the fixed ratio as 1:2.
E. Before pressing enter, be sure that you see the ghost of the transformation. Does the ghost correspond with the ratio of 1:2? In what way?
F. Press Dilate in the pop-up menu.
G. Select the center of dilation, *C*, and move it around the sketch. What happens to the image as you move the center throughout the sketch? In particular, where is it with respect to triangle EWS? Explain.
H. What happens when the center is moved to one of the vertices of *EWS*? Explain.
I. What happens when the center is moved to the interior of *EWS*? Explain.

1. If the lengths of the midsegments of a triangle are 3, 4, and 5, what is the perimeter of the triangle?

2. During the time that they are moving the kitchen, the dining hall will pack our plates in square boxes with a perimeter of 36 inches. If each plate fits snugly in the box, what is the circumference of each plate?

3. (Continuation) Each dining hall saucer has a circumference of 12.57 in. Can four saucers fit side by side in the same square box? If so, how many inches will be left for padding?

4. Graph the circle whose equation is $x^2 + y^2 = 64$. What is its radius? What do the equations $x^2 + y^2 = 1$, $x^2 + y^2 = 40$, and $x^2 + y^2 = k$ all have in common? How do they differ?

1. Let $A = (0, 5)$, $B = (-2, 1)$, $C = (6, -1)$, and $P = (12, 9)$. Let A', B', and C' be the midpoints of segments PA, PB, and PC, respectively. After you make a diagram, identify the center and the magnitude of the dilation that transforms triangle ABC onto $A'B'C'$.

2. Taylor lets out 120 meters of kite string then wonders how high the kite has risen. Taylor is able to calculate the answer after using a protractor to measure the 63-degree angle of elevation that the string makes with the ground. How high is the kite, to the nearest meter? What (unrealistic) assumptions did you make in answering this question?

3. Find the sine of a 45-degree angle. Use your calculator *only to check your answer*.

4. A triangle that has a 5-inch and a 6-inch side can be similar to a triangle that has a 4-inch and an 8-inch side. Find all possible examples. Check that your examples really *are* triangles.

5. Using GSP: Let $A = (1, 5)$, $B = (3, 1)$, $C = (5, 4)$, $A' = (5, 9)$, $B' = (11, -3)$, and $C' = (17, 6)$. Show that there is a dilation that transforms triangle ABC onto triangle $A'B'C'$. In other words, find the dilation center and the scale factor.

6. (Continuation) Calculate the areas of triangles ABC and $A'B'C'$. Are your answers related in a predictable way?

7. If the central angle of a slice of pizza is 36 degrees, how many pieces are in the pizza?

8. (Continuation) A 12 inch pizza is evenly divided into 8 pieces. What is the length of the crust of one piece?

9. The vertices of triangle ABC are $A = (-5, -12)$, $B = (5, -12)$, and $C = (5, 12)$. Confirm that the circumcenter of ABC lies at the origin. What is the equation for the *circumscribed circle*?

10. If the sides of a triangle are 13, 14, and 15 cm long, then the altitude drawn to the 14-cm side is 12 cm long. How long are the other two altitudes? Which side has the longest altitude?

11. (Continuation) How long are the altitudes of the triangle if you double the lengths of its sides?

12. Let $A = (6, 0)$, $B = (0, 8)$, $C = (0, 0)$. In triangle ABC, let F be the point of intersection of the altitude drawn from C to side AB.
    (a) Explain why the angles of triangles ABC, CBF, and ACF are the same.
    (b) Find coordinates for F and use them to calculate the exact lengths FA, FB, and FC.
    (c) Compare the sides of triangle ABC with the sides of triangle ACF. What do you notice?

13. What happens to the area of a triangle if its dimensions are doubled?

1. A rectangular sheet of paper is 20.5 cm wide. When it is folded in half, with the crease running parallel to the 20.5-cm sides, the resulting rectangle is the same shape as the unfolded sheet. Find the length of the sheet, to the nearest tenth of a cm. (In Europe, the shape of notebook paper is determined by this similarity property).

2. *SAS Similarity*. Use your protractor to carefully draw a triangle that has a 3-cm side, a 4-cm side, and whose *included angle* is 40 degrees. Construct a second triangle that has a 7.5-cm side, a 10-cm side, and whose included angle is also 40 degrees. Measure the remaining parts of these triangles. Could you have anticipated the results? Explain.

3. A regular polygon is inscribed in a circle. What happens to the regular polygon as the number of sides increases?

4. Sketch the circle whose equation is $x^2 + y^2 = 100$. Using the same system of coordinate axes, graph the line $x + 3y = 10$, which should intersect the circle twice – at $A = (10, 0)$ and at another point B in the second quadrant. Estimate the coordinates of B. Now use algebra to find them exactly. Segment AB is called a *chord* of the circle.

5. (Continuation) Find coordinates for a point C on the circle that makes chords AB and AC have equal length.

6. What is the radius of the smallest circle that surrounds a 5-by-12 rectangle?

7. Without doing any calculation, what can you say about the tangent of a k-degree angle, when k is a number between 90 and 180? Explain your response, then check with your calculator.

8. A right triangle has a 123-foot hypotenuse and a 38-foot leg. To the nearest tenth of a degree, what are the sizes of its acute angles?

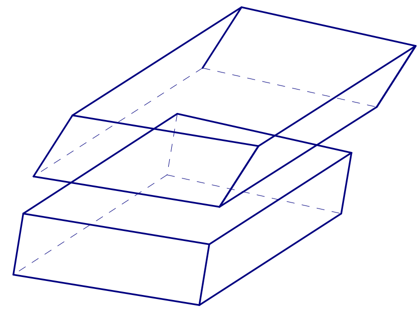
9. Ask your calculator for the sine of a 56-degree angle, then for the cosine of a 34-degree angle. Ask your calculator for the sine of a 23-degree angle, then for the cosine of a 67-degree angle. The word *cosine* is an abbreviation of *sine of the complement*. Explain the terminology.

10. (Continuation) How can you represent the cosine of an angle in terms of a ratio?

11. The line $y = x + 2$ intersects the circle $x^2 + y^2 = 10$ in two points. Call the third quadrant point R and the first-quadrant point E, and find their coordinates. Let D be the point where the line through R and the center of the circle intersects the circle again. The chord DR is an example of a *diameter*. Show that triangle RED is a right triangle.

12. To the nearest tenth of a degree, find the angles of the triangle with vertices $(0, 0)$, $(6, 3)$, and $(1, 8)$. Use your protractor to *check* your calculations, and explain your method.

1. Playing cards measure 2.25 inches by 3.5 inches. A full deck of fifty-two cards is 0.75 inches high. What is the volume of a deck of cards? If the cards were uniformly shifted (turning the bottom illustration into the top illustration), would this volume be affected?



2. In a right triangle, the 58-cm hypotenuse makes a 51-degree angle with one of the legs. To the nearest tenth of a cm, how long is that leg? Once you have the answer, find some other ways to calculate the length of the other leg. They should all give the same answer, of course.

3. An equilateral triangle ABC is inscribed in a circle centered at *O*. The portion of the circle that lies above chord *AB* is called an *arc*. If $\overset{\frown}{AB} = \overset{\frown}{BC} = \overset{\frown}{AC}$, what is the measure of $\overset{\frown}{AB}$? $\overset{\frown}{AB}$ is called a *minor arc* and $\overset{\frown}{ACB}$ is called a *major arc*. Why do you think they are called this? How are $\overset{\frown}{AB}$ and $\overset{\frown}{ACB}$ related?

4. (Continuation) *Some Terminology:* A *central angle* is an angle whose vertex is at the center of a circle and whose sides are radii. What is the measure of angle *AOB*? What is the relationship between a central angle and the arc it intercepts?

5. Draw a large triangle *ABC* and mark *D* on segment *AC* so that the ratio *AD: DC* is equal to $3:4$. Mark any point *P* on segment *BD*.
   (a) Find the ratio of the area of triangle *BAD* to the area of triangle *BCD*.
   (b) Find the ratio of the area of triangle *PAD* to the area of triangle *PCD*.
   (c) Find the ratio of the area of triangle *BAP* to the area of triangle *BCP*.

6. What is the angular size of an arc that a diameter intercepts? This arc is called a *semicircle*?

7. If the ratio of the areas of two triangles is 18 : 8, what is the ratio of a pair of two corresponding sides?

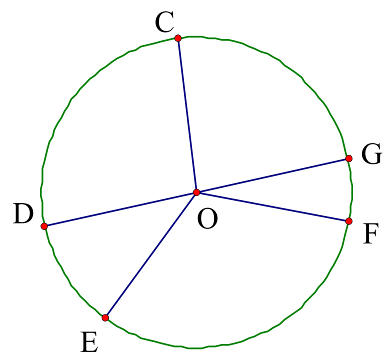
8. The vertices of a square with sides parallel to the coordinate axes lie on the circle of radius 5 whose center is at the origin. Find coordinates for the four vertices of this square.

9. Let *A* = (0, 1), *B* = (7, 0), *C* = (3, 7), and *D* = (0, 6). Find the areas of triangles *ABC* and *ADC*, which share side *AC*. Calculate the ratio of areas *ABC: ADC*. If you were to calculate the distances from *B* and *D* to the line *AC*, how would they compare? Explain your reasoning, or else calculate the two distances to confirm your prediction.

10. Draw a circle and label one of its diameters *AB*. Choose any other point on the circle and call it *C*. What can you say about the size of angle *ACB*? Does it depend on which *C* you chose? Justify your response.

1. A square *pyramid* is a pyramid with a square base and four triangular lateral faces. The slant height is the distance from the vertex of the pyramid along a *lateral face* to the midpoint of a base edge. If the slant height is 10 and an edge of the square is 12, what is the altitude of this pyramid?

2. Circle O has diameter DG and central angles $COG = 86$, $DOE = 25$, and $FOG = 15$. Find minor arcs $\overset{\frown}{CG}$, $\overset{\frown}{CF}$, $\overset{\frown}{EF}$ and major arc $\overset{\frown}{DGF}$.



3. Draw a circle. Draw two chords of unequal length. Which chord is closer to the center of the circle? What can be said about the *intercepted arcs*?

4. Draw two circles with radii of different lengths. Pick a length. Draw a chord in each circle of this length. What can be said about the respective intercepted arcs?

5. *The area of a circle:* A regular hundred-sided polygon with side length of b=2 is inscribed in a circle with radius of 32 (do not attempt to draw this polygon!)
   (a) Show that the perimeter of the polygon is approximately equal to the circumference of the circle.
   (b) Show that the height of a central triangle of the polygon is approximately equal to the radius of the circle.
   (c) You can write the area of the polygon as $A_{polygon} = 100 \cdot$ area of a central triangle. Justify rewriting this expression as $A_{polygon} = 100 \cdot b \cdot \frac{1}{2} \cdot h$.
   (d) Now substitute $2\pi r$ for $100 \cdot b$. Why can you do this?
   (e) Then substitute $\frac{1}{2} r$ for $\frac{1}{2} b$. Why can you do this?
   (f) Simplify your expression for the area of the circle.

6. If two chords in the same circle have the same length, then their minor arcs have the same length, too. True or false? Explain. What about the converse statement? Is it true? Why?

7. Suppose that MP is a diameter of a circle centered at O, and Q is any other point on the circle. Draw the line through O that is parallel to MQ, and let R be the point where it meets minor arc PQ. Prove that R is the midpoint of minor arc PQ.

8. A regular pentagon can be dissected into 5 isosceles triangles whose vertex angle is at the center of the pentagon. The height of the triangles is 10 cm. Find the area of this pentagon.

9. The circle $x^2 + y^2 = 25$ goes through $A = (5, 0)$ and $B = (3, 4)$. To the nearest tenth of a degree, find the size of the minor arc AB.

10. An equilateral triangle is inscribed in the circle of radius 1 centered at the origin (the *unit circle*). If one of the vertices is (1, 0), what are the coordinates of the other two? The three points divide the circle into three arcs; what are the angular sizes of these arcs?

1. The sides of a triangle are found to be 10 cm, 14 cm, and 16 cm long, while the sides of another triangle are found to be 15 in, 21 in, and 24 in long. On the basis of this information, what can you say about the angles of these triangles? Is it possible to *calculate* their sizes?

2. The points *A*, *P*, *Q*, and *B* appear in this order on a line, so that *AP: PQ* = 2: 3 and *PQ: QB* = 5: 8. Find whole numbers that are proportional to *AP: PQ: QB*.

3. In mathematical discussion, a *right prism* is defined to be a solid figure that has two parallel, congruent polygonal bases, and rectangular *lateral faces*. How would you find the volume of such a figure? Explain your method.

4. On a circle whose center is *O*, using your protractor, mark points *P* and *A* so that minor arc *PA* is a 46-degree arc. What does this tell you about angle *POA*? Extend *PO* to meet the circle again at *T*. What is the size of angle *PTA*? This angle is *inscribed* in the circle, because its vertex is on the circle. The arc *PA* is *intercepted* by the angle *PTA*. Make a conjecture about arcs intercepted by inscribed angles.

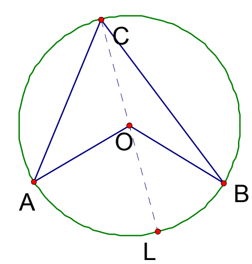
5. The area of triangle *ABC* is 231 square inches, and point *P* is marked on side *AB* so that *AP*: *PB* = 3: 4. What are the areas of triangles *APC* and *BPC*?

6. In triangle *ABC*, it is given that angle *BCA* is right. Let $a = BC$, $b = CA$, and $c = AB$. Using *a*, *b*, and *c*, express the sine, cosine, and tangent ratios of acute angles *A* and *B*.

7. The sine of a 38-degree angle is some number *r*. Without using your calculator, you should be able to identify the angle size whose cosine is the same number *r*.

8. Given *SSS* information about an isosceles triangle, describe the process you would use to calculate the sizes of its angles.

9. *The Star Trek Theorem*: You have found that an inscribed angle is half the measure of the arc that it intercepts.
   (a) Given a circle centered at O, let A, B and C be points on the circle such that arc AC is not equal to arc BC and CL is a diameter. Why must triangles AOC and AOB be isosceles?
   (b) State the pairs of angles that must be congruent in these isosceles triangles.
   (c) Using the Exterior Angle Theorem, find expressions for the measures of $\angle AOL$ and $\angle BOL$.
   (d) Based on your statement in part c, explain the statements $\angle ACL = \frac{1}{2}(\angle AOL)$ and $\angle OCB = \frac{1}{2}(\angle BOL)$.
   (e) Now find an expression for $\angle ACB$ and simplify to prove that it equals $\frac{1}{2}\angle AOB$.



10. If *P* and *Q* are points on a circle, then the center of the circle must be on the perpendicular bisector of chord *PQ*. Explain. Which point on the chord is closest to the center? Why?

1. Given that triangle *ABC* is similar to triangle *PQR*, write the three-term proportion that describes how the six sides of these figures are related.

2. A hexagon is inscribed in a circle of radius 5. Find the area of the hexagon.

3. A hexagon has an inscribed circle of radius 4. Find the area of the hexagon.

4. Draw a circle with a 2-inch radius, mark four points randomly (not evenly spaced) on it, and label them consecutively *G*, *E*, *O*, and *M*. Measure angles *GEO* and *GMO*. Could you have predicted the result? Name another pair of angles that would have produced the same result.

5. A circular park 80 meters in diameter has a straight path cutting across it. It is 24 meters from the center of the park to the closest point on this path. How long is the path?

6. Show that the lines $y = 2x - 5$ and $-2x + 11y = 25$ create chords of equal length when they intersect the circle $x^2 + y^2 = 25$. Make a large diagram, and measure the inscribed angle formed by these chords. Describe two ways of calculating its size to the nearest 0.1 degree. What is the angular size of the arc that is intercepted by this inscribed angle?

7. A triangle has a 3-inch side, a 4-inch side, and a 5-inch side. The altitude drawn to the 5-inch side cuts this side into segments of what lengths?

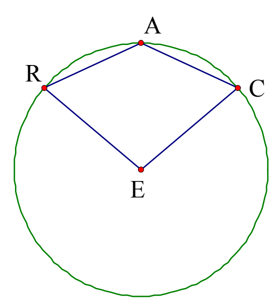
8. A chord 6 cm long is 2 cm from the center of a circle. How long is a chord that is 1 cm from the center of the same circle?

9. By using the triangle whose sides have lengths 1, $\sqrt{3}$, and 2, you should be able to write non-calculator expressions for the sine, cosine, and tangent of a 30-degree angle. Do so. You can use your calculator to check your answers, of course.

10. Triangle *ABC* is inscribed in a circle. Given that *AB* is a 40-degree arc and *ABC* is a 50-degree angle, find the sizes of the other arcs and angles in the figure.

11. Suppose that chords *AB* and *BC* have the same lengths as chords *PQ* and *QR*, respectively, with all six points belonging to the same circle (they are *concyclic*). Is this enough information to conclude that chords *AC* and *PR* have the same length? Explain.

12. The figure at right shows points *C*, *A*, and *R* marked on a circle centered at *E*, so that chords *CA* and *AR* have the same length, and so that major arc *CR* is a 260-degree arc. Find the angles of quadrilateral *CARE*. What is special about the sizes of angles *CAR* and *ACE*?



13. Find all the angles in a 5-12-13 triangle.

1. A trapezoid has two 65-degree angles and 8-inch and 12-inch parallel sides. How long are the non-parallel sides? What is the area enclosed by this figure?

2. Two circles of radius 10 cm are drawn so that their centers are 12 cm apart. The two points of intersection determine a *common chord*. Find the length of this chord.

3. Find a triangle two of whose angles have sizes $\text{TAN}^{-1}(1.5)$ and $\text{TAN}^{-1}(3)$. Answer this question either by giving coordinates for the three vertices, or by giving the lengths of the three sides. To the nearest 0.1 degree, find the size of the third angle in your triangle.

4. What is the sine of the angle whose tangent is 2? First find an answer *without* using your calculator (draw a picture) then use your calculator to check.

5. Find the area of a regular 36-sided polygon inscribed in a circle of radius 20 cm.

6. You are at the scenic overlook at Thatcher Park looking through the panoramic viewer that is looking straight ahead. How far must you rotate the viewer directly downward to see the Egg in Albany, which you know to be 30 miles away. The scenic overlook is two-tenths of a mile high.

7. Can the diagonals of a kite bisect each other?

8. The area of an equilateral triangle is $100\sqrt{3}$ square inches. How long are its sides?

9. Points *E*, W, and *S* are marked on a circle whose center is *N*. In quadrilateral *NEWS*, angles *S* and *W* are found to be 54° and 113°, respectively. What are the other two angles?

10. The points $A = (0, 13)$ and $B = (12, 5)$ lie on a circle whose center is at the origin. Show that the perpendicular bisector of *AB* goes through the origin.

11. The areas of two similar triangles are 24 square cm and 54 square cm. The smaller triangle has a 6-cm side. How long is the corresponding side of the larger triangle?

12. Find the perimeter of a regular 36-sided polygon inscribed in a circle of radius 20 cm.

13. When two circles have a common chord, their centers and the endpoints of the chord form a quadrilateral. What kind of quadrilateral? What special property do its diagonals have?

14. Given that θ (*Greek* "theta") stands for the degree size of an acute angle, fill in the blank space between the parentheses to create a true statement: $\sin\theta = \cos(\ )$.

15. If corresponding sides of two similar triangles are in a 3: 5 ratio, what is the ratio of the areas of these triangles?

1. Let $P = (-25, 0)$, $Q = (25, 0)$, and $R = (-24, 7)$.
   (a) Find an equation for the circle that goes through P, Q, and R.
   (b) Find at least two ways of showing that angle PRQ is right.
   (c) Find coordinates for three other points R that would have made angle PRQ right.

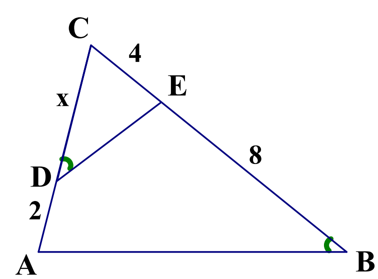
2. How much evidence is needed to be sure that two triangles are similar?

3. Trapezoid $ABCD$ has parallel sides AB and CD, of lengths 8 and 16, respectively. Diagonals AC and BD intersect at E, and the length of AC is 15. Find the lengths of AE and EC.

4. Let $A = (0, 0)$, $B = (4, 0)$, and $C = (4, 3)$. Mark point D so that ACD is a right angle and DAC is a 45-degree angle. Find coordinates for D. Find the tangent of angle DAB.

5. A regular octagon has a perimeter of 64. Find its area.

6. Two circles have a 24-cm common chord, their centers are 14 cm apart, and the radius of one of the circles is 13 cm. Make an accurate drawing, and find the radius for the second circle in your diagram. Are there other possible answers?



7. Refer to the figure, in which angles ABE and CDE are equal in size and various segments have been marked with their lengths. Find x.

8. A kite has an 8-inch side and a 15-inch side, which form a right angle. Find the length of the diagonals of the kite.

9. Mark points A and C on a clean sheet of paper, then spend a minute or so drawing rectangles $ABCD$. What do you notice about the configuration of points B and D?

10. What is the radius of the circumscribed circle for a triangle whose sides are 15, 15, and 24 cm long?

11. A *cyclic* quadrilateral is a quadrilateral whose vertices are points on a circle. Draw a cyclic quadrilateral $SPAM$ in which the size of angle SPA is 110 degrees. What is the size of angle AMS? Would your answer change if M were replaced by a different point on major arc SA?

12. Let $A = (0, 0)$ and $B = (0, 8)$. Plot several points P that make APB a 30-degree angle. Use a protractor and be prepared to report coordinates for your points. Formulate a conjecture about the configuration of all such points.

13. If A = (3, 1), B = (3, 4), and C = (8, 1) find the measure of angle B.

14. A 20-inch chord is drawn in a circle with a 12-inch radius. What is the *angular size* of the minor arc of the chord? What is the *length* of the arc, to the nearest tenth of an inch?

1. When you first walk into Weaver there are two large regular octagonal pillars. The edges are 6.5 in and they are 9 feet tall. How much granite was needed to build these pillars?

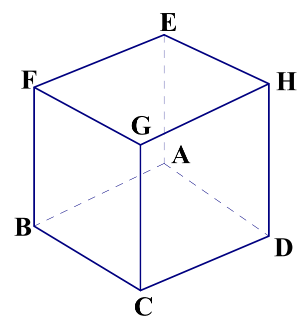
2. Quadrilateral *WISH* is *cyclic*. Diagonals *WS* and *HI* intersect at *K*. Given that arc *WI* is 100 degrees and arc *SH* is 80 degrees, find the sizes of as many angles in the figure as you can. Note: *K* is not the center of the circle.

3. Triangle *ABC* has *P* on *AC*, *Q* on *AB*, and angle *APQ* equal to angle *B*. The lengths *AP* = 3, *AQ* = 4, and *PC* = 5 are given. Find the length of *AB*.

4. Draw the line $y = 2x - 5$ and the circle $x^2 + y^2 = 5$. Use algebra to show that these graphs touch at only one point. It is customary to say that a line and a circle are *tangent* if they have exactly one point in common.

5. (Continuation) Find the slope of the segment that joins the point of tangency to the center of the circle and compare your answer with the slope of the line $y = 2x - 5$. What do you notice?

6. A kite has a 5-inch side and a 7-inch side. One of the diagonals is bisected by the other. The bisecting diagonal has length 8 inches. Find the length of the bisected diagonal.



7. Given that *ABCDEFGH* is a cube (shown at right), what is significant about the square pyramids *ADHEG*, *ABCDG*, and *ABFEG*?

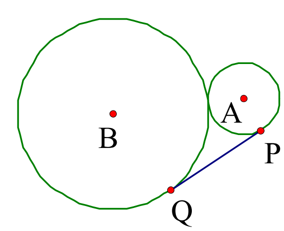
8. Quadrilateral *BAKE* is cyclic. Extend *BA* to a point *T* outside the circle, thus producing the exterior angle *KAT*. Why do angles *KAT* and *KEB* have the same size?

9. Drawn in a circle whose radius is 12 cm, chord *AB* is 16 cm long. Calculate the angular size of minor arc *AB*.

10. Show that the line $y = 10 - 3x$ is tangent to the circle $x^2 + y^2 = 10$. Find an equation for the line perpendicular to the tangent line at the point of tangency. Show that this line goes through the center of the circle.

11. Let *A* = (4, 6), *B* = (6, 0), and *C* = (9, 9). Find the size of angle *BAC*.

12. A circle with a 4-inch radius is centered at *A* and a circle with a 9-inch radius is centered at *B*, where *A* and *B* are 13 inches apart. There is a segment that is tangent to the small circle at *P* and to the large circle at *Q*. It is a common external tangent of the two circles. What kind of quadrilateral is *PABQ*? What are the lengths of its sides?

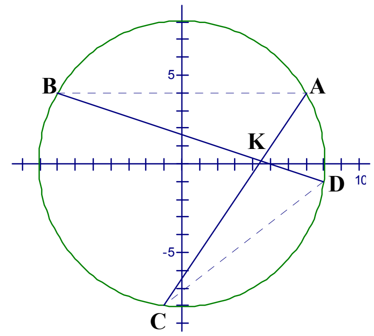


1. The Robison track is shaped like a rectangle with a semicircle on each of the shorter sides. The distance around the track is one-quarter mile. The straightaway is twice as long as the width of the field. What is the area of the field enclosed by the track to the nearest square foot?

2. Segment *AB*, which is 25 inches long, is the diameter of a circle. Chord *PQ* meets *AB* perpendicularly at *C*, where *AC* = 16 in. Find the length of *PQ*.

3. Prove that the arcs between any two parallel chords in a circle must be the same size.

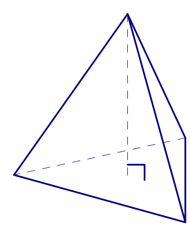
4. *Crossed Chords*. Verify that *A* = (7, 4), *B* = (−7, 4), *C* = (-1, -8), and *D* = (8, -1) all lie on a circle centered at the origin. Let *K* be the intersection of chords *AC* and *BD*. By measuring angles, convince yourself that triangles *KAB* and *KDC* are similar. Could this have been predicted? Write out the three-term proportion. To three significant figures, what is the ratio of similarity?



5. (Continuation) Explain why triangles *KAD* and *KBC* are also similar. What is the ratio of similarity? Is it the same as for the other pair of similar triangles?

6. *Two Tangent Theorem*. From any point *P* outside a given circle, there are two lines through *P* that are tangent to the circle. Explain why the distance from *P* to one of the points of tangency is the same as the distance from *P* to the other point of tangency. What special quadrilateral is formed by the center of the circle, the points of tangency, and *P*?

7. The altitude of a regular triangular pyramid is the segment connecting a vertex to the centroid of the opposite face. A regular triangular pyramid has edges of length 6 in. How tall is such a pyramid, to the nearest hundredth of an inch?



8. A 72-degree arc *AB* is drawn in a circle of radius 8 cm. How long is chord *AB*?

9. Find the perimeter of a regular 360-sided polygon that is inscribed in a circle of radius 5 inches. If someone did not remember the formula for the circumference of a circle, how could that person use a calculator's trigonometric functions to find the circumference of a circle with a 5-inch radius?

10. The segments *GA* and *GB* are tangent to a circle with center *O* at *A* and *B*, and *AGB* is a 60-degree angle. Given that $GA = 12\sqrt{3}$ cm, find the distance *GO*. Find the distance from *G* to the nearest point on the circle.

11. A circle goes through the points *A*, *B*, *C*, and *D* consecutively. The chords *AC* and *BD* intersect at *P*. Given that *AP* = 6, *BP* = 8, and *CP* = 3, how long is *DP?*

1. A circle T has two tangents that intersect at 54° at point M. The points of tangency are A and H. What is the angular size of $\overset{\frown}{AH}$?

2. The Great Pyramid at Giza was originally 483 feet tall, and it had a square base that was 756 feet on a side. It was built from rectangular stone blocks measuring 7 feet by 7 feet by 15 feet. Such a block weighs seventy tons. Approximately how many tons of stone were used to build the Great Pyramid? The volume of a pyramid is one third the base area times the height.

3. What is the difference between the measure of an arc and the length of an arc?

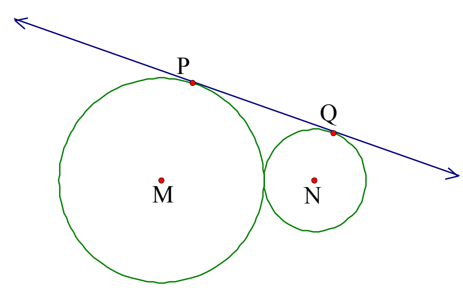
4. A piece of a broken circular gear is brought into a metal shop so that a replacement can be built. A ruler is placed across two points on the rim, and the length of the chord is found to be 14 inches. The distance from the midpoint of this chord to the nearest point on the rim is found to be 4 inches. Find the radius of the original gear.

5. In a group of 12 students, only 4 of them like olives on their pizza. If they are sharing a 16-in pizza what is the area of the part the pizza covered with olives?

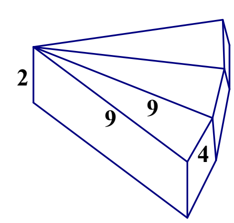
6. A triangle that has a 50-degree angle and a 60-degree angle is inscribed in a circle of radius 25 inches. The circle is divided into three arcs by the vertices of the triangle. To the nearest tenth of an inch, find the lengths of these three arcs.

7. Stacy wants to decorate the side of a cylindrical can by using a rectangular piece of paper and wrapping it around the can. The paper is 21.3 cm by 27.5 cm. Find the two possible diameters of the cans that Stacy could use. (Assume the paper fits exactly).

8. PQ is tangent to circles $\odot M$ and $\odot N$. $\odot M$ and $\odot N$ are externally tangent.
(a) If $\odot M$ and $\odot N$ are not congruent; what kind of quadrilateral is $MNPQ$?
(b) If $\odot M$ and $\odot N$ are congruent, what kind of quadrilateral is $MNPQ$.



9. A wedge of cheese is 2 inches tall. The triangular base of this right prism has two 9-inch edges and a 4-inch edge. Several congruent wedges are arranged around a common 2-inch segment, as shown. How many wedges does it take to complete this wheel? What is the volume of the wheel, to the nearest cubic inch?



10. Chords AB and CD intersect at P. Given $AP = 16$, $BP = 9$, and $CP = 8$, find DP.

11. The segments GA and GB are tangent to a circle at A and B, and AGB is a 48-degree angle. Given that $GA = 12$ cm, find the distance from G to the nearest point on the circle.

1. A 16.0-inch chord is drawn in a circle whose radius is 10.0 inches. What is the angular size of the minor arc of this chord? What is the length of the arc, to the nearest tenth of an inch?

2. What is the area enclosed by a circular *sector* whose radius is r and *arc length* is s?

3. The line $x + 2y = 5$ divides the circle $x^2 + y^2 = 25$ into two arcs. Find the lengths of the arcs.

4. (Continuation) The region enclosed by the circle is also divided into two parts. Find the area of the smaller one.

5. The equation of a circle is $x^2 + y^2 = 50$, find the area of the circle.

6. If the area of a circle centered at the origin is 40π, write the equation for this circle.

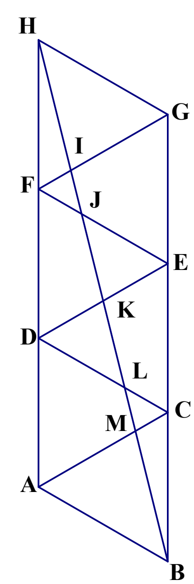
7. Is the length of a circular arc proportional to the length of its chord? Explain your answer.

8. Write the equation of the circle that passes through the vertices of the triangle defined by (-1, -7), (5, 5), (7, 1).

9. Alex's dog, Fluffy, is tied with a 20 ft rope to the center of the back wall of the shed, which has dimensions of 14 ft by 18 ft. If the back wall is the longer wall, over what area can Fluffy play, to the nearest square foot? Would your answer change if the back wall was 14 ft instead?

10. All triangles and rectangles have circumscribed circles. Is this true for all kites, trapezoids, and parallelograms? Explain. Which quadrilaterals have circumscribed circles? Explain.

11. The figure at right is built by joining six equilateral triangles *ABC*, *ACD*, *CDE*, *DEF*, *EFG*, and *FGH*, all of whose edges are 1 unit long. It is given that *HIJKLMB* is straight.
(a) There are five triangles in the figure that are similar to *CMB*. List them, making sure that you match corresponding vertices.
(b) Find the lengths of *CM* and *EK*.
(c) List the five triangles that are similar to *AMB*.
(d) Find the lengths of *CL*, *HI*, *IJ*, and *JK*.



12. Pyramid *TABCD* has a 20-cm square base *ABCD*. The edges that meet at *T* are 27 cm long. Make a diagram of *TABCD*, showing *F*, the point of *ABCD* closest to *T*. To the nearest 0.1 cm, find the height *TF*. Find the volume of *TABCD*, to the nearest cm^3.

13. (Continuation) Find the slant height of pyramid TABCD.

14. (Continuation) Let *K*, *L*, *M*, and *N* be the points on *TA*, *TB*, *TC*, and *TD*, respectively, that are 18 cm from *T*. What can be said about polygon *KLMN*? Explain.

1. Two of the tangents to a circle meet at Q, which is 25 cm from the center. The circle has a 7-cm radius. To the nearest tenth of a degree, find the angle formed at Q by the tangents.

2. Four points on a circle divide it into four arcs whose sizes are 52 degrees, 116 degrees, 100 degrees, and 92 degrees, in consecutive order. The four points determine two intersecting chords. Find the sizes of the angles formed by the intersecting chords.

3. Can a circle always be drawn through three given points? If so, describe a procedure for finding the center of the circle. If not, explain why not.

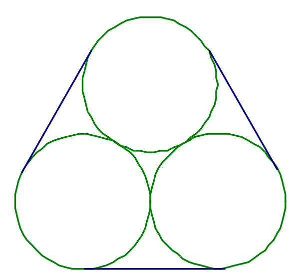
4. Which is the better (tighter) fit: A round peg in a square hole or a square peg in a round hole?

5. From the top of Mt Washington, which is 6288 feet above sea level, how far is it to the horizon? Assume that the Earth has a 3962-mile radius, and give your answer to the nearest mile.

6. What is the minimum amount of wrapping paper needed to wrap a box with dimensions 20 cm by 10 cm by 30 cm?

7. A paper towel tube has a diameter of 1.7 inches and a height of 11 inches. If the tube were cut and unfolded to form a rectangle, what would be the area of the rectangle?

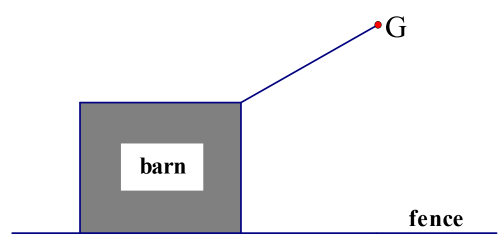
8. Find the area of a kite whose longer diagonal is divided into two parts that are 4 and 12 and whose shorter side is 5.



9. The figure shows three circular pipes, all with 12-inch diameters, that are strapped together by a metal band. How long is the band?

10. (Continuation) Suppose that four pipes are strapped together with a snugly-fitting metal band. How long is the band?

11. The lateral edges of a regular hexagonal pyramid are all 20 cm long, and the base edges are all 16 cm long. To the nearest cm^3, what is the volume of this pyramid? To the nearest square cm, what is the combined area of the base and six lateral faces?



12. Find the total grazing area of the goat G represented in the figure (a top view) shown at right. The animal is tied to a corner of a $40' \times 40'$ barn, by an $80'$ rope. One of the sides of the barn is extended by a fence. Assume that there is grass everywhere except inside the barn.

13. *Surface area of a Sphere*: The surface area of a sphere is found using the formula $4\pi r^2$. Find the surface area of the Earth, given that its diameter is 7924 miles.

1. For any pyramid, the volume is $\frac{1}{3}$ □base area□height. A cone is a pyramid with a circular base. Find the volume of a cone with a slant height of 13 and a diameter of 10.

2. A sector of a circle is enclosed by two 12.0-inch radii and a 9.0-inch arc. Its perimeter is therefore 33.0 inches. What is the area of this sector, to the nearest tenth of a square inch? What is the central angle of the sector, to the nearest tenth of a degree?

3. (Continuation) There is another circular sector – part of a circle of a different size – that has the same 33-inch perimeter and that encloses the same area. Find its central angle, radius, and arc length, rounding the lengths to the nearest tenth of an inch.

4. The radius of the Sun is 109 times the radius of the Earth. Find the surface area of the Sun.

5. The radius of a circular sector is r. The central angle of the sector is θ. Write formulas for the arc length and the perimeter of the sector.

6. Suppose that the lateral faces VAB, VBC, and VCA of triangular pyramid $VABC$ all have the same height drawn from V. Let F be the point in base ABC that is closest to V, so that VF is the altitude of the pyramid. Show that F is one of the special points of triangle ABC.

7. Schuyler has made some glass prisms to be sold as window decorations. Each prism is four inches tall, and has a regular hexagonal base with half-inch sides. They are to be shipped in cylindrical tubes that are 4 inches tall. What radius should Schuyler use for the tubes? Once a prism is inserted into its tube, what volume remains for packing material?

8. A conical cup has a 10-cm diameter and is 12 cm deep. How much can this cup hold?

9. (Continuation) Water in the cup is 6 cm deep. What percentage of the cup is filled?

10. Imagine the interior of a sphere can be approximated by numerous cones, each with a base area, B, height of r, and vertex at the center of the sphere. What formula do you already know that describes the sum of the base areas?

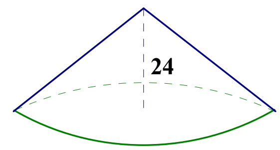
11. (Continuation) Given this approximation of a sphere, develop a formula for its volume.

12. A sphere of ice cream is placed on an ice cream cone. Both have a diameter of 8 cm. The height of the cone is 12 cm. Will all the ice cream, if pushed down into the cone, fit?

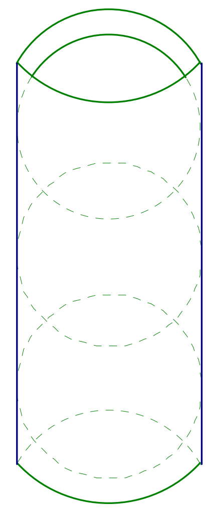
13. What is the angle formed by the hands on a clock at 10:20? Hint: It is not 180 degrees.

14. The base radius of a cone is 6 inches, and the cone is 8 inches tall. To the nearest square inch, what is the area of the lateral surface of the cone?

1. The highway department keeps its sand in a conical storage building that is 24 feet high and 64 feet in diameter. To estimate the cost of painting the building, the lateral surface area of the cone is needed. To the nearest square foot, what is the area?



2. If the surface area of a sphere equals the surface area of a cube, what is the ratio of the volume of the sphere to the volume of the cube?



3. Three tennis balls fit snugly inside a cylindrical can. What percent of the available space inside the can is occupied by the balls?

4. The areas of two circles are in the ratio of 50: 32. What is the ratio of their radii?

5. In an effort to make their product seem like a better bargain, the Chock-a-Lot candy company increased the size of their chocolate balls, from a 2-cm diameter to a 3-cm diameter, without increasing the price. In fact, the new balls still contain the same amount of chocolate, because they are *hollow spherical shells*, while the 2-cm balls are solid chocolate. How thick are the spherical chocolate shells that Chock-a-Lot is now selling?

6. The area of a sector of a circle with radius 12 is 16π cm^2. What is the central angle of this sector?

7. Find the perimeter of the semicircle with radius 10.

8. A squash ball fits snugly inside a cubical box whose edges are 4 cm long. Guess the percentage of the box's volume that the ball occupies, then calculate that percentage. (This is an example of a *sphere inscribed in a cube*.)

9. There is a park, 27 feet wide, that is between two buildings whose heights are 123 ft and 111 ft. Two Emma Willard teachers, Dr. Naeher and Ms. Snyder are standing on top of the shorter building looking at a rare bird perched on top of the taller building. If Dr. Naeher is 74 inches tall and Ms. Snyder is 62 inches tall who has the smaller angle of elevation while looking at the bird? Explain your answer.

10. A conical cup is 64/125 full of liquid. What is the ratio of the depth of the liquid to the depth of the cup? Conical cups appear fuller than cylindrical cups – explain why.

11. As a spherical gob of ice cream that once had a 2-inch radius melts, it drips into a cone of the same radius. The melted ice cream exactly fills the cone. What is the height of the cone?

12. Two similar triangles have medians in a ratio of 5: 6, what is the ratio of their areas?

1. Find the volume of material that is needed to form a spherical shell whose outer radius is 6.0 inches and whose thickness is 0.01 inch. Use your answer to estimate the surface area of the 6-inch sphere.

2. A 10 cm tall cylindrical glass 8 cm in diameter is filled to 1 cm from the top with water. If a gold ball 4 cm in diameter is dropped into the glass, will the water overflow?

3. An *annulus* is defined as the region lying between two *concentric* circles. If the diameter of the larger circle is 20 in and the radius of the smaller circle is 8, find the area of the annulus.

4. A Reese's Big Cup has a diameter of two inches and 0.8 inches. A Reese's bar has dimensions 4 inches by 0.8 inches by 0.5 inches. Using approximations, which candy has more peanut butter?

5. (Continuation) Assuming a uniform chocolate thickness, which candy has more chocolate?

6. The altitudes of two similar triangles are 6 cm and 9 cm. If the area of the larger triangle is 36 cm^2, what is the area of the smaller triangle?

7. Ice cream scoops are often 1.25 inches in diameter. How many scoops should you get from a half-gallon of ice cream? A half-gallon container can be approximated by a cylinder with a diameter of 4.8 inches and a height of 6.5 inches.

8. A spherical globe, 12 inches in diameter, is filled with spherical gumballs, each having a 1-inch diameter. Estimate the number of gumballs in the globe, and explain your reasoning.

9. Seventy percent of the Earth's surface is covered in water. Find the approximate surface area of the Earth that is dry land.

10. Given triangle *EWS* defined by *E*(5, 5), *W*(4, -8), and *S*(-6, 6), write the equation of the median from point *E* to *WS*. How far is it from point *E* to the centroid?

11. The corresponding edges of two similar triangular prisms are in the ratio 3: 5. What is the ratio of their surface areas? What is the ratio of their volumes?

12. The sum of the lengths of the two bases of a trapezoid is 22 cm and its area is 946 cm^2. Find the height of this trapezoid.

13. The volumes of two similar hexagonal prisms are in the ratio of 8: 125. What is the ratio of their heights?

14. Find the point that is equidistant from the points (0, 4), (2, 3), and (5, 9).

1. The base of a pyramid is the regular polygon *ABCDEFGH*, which has 14-inch sides. All eight of the pyramid's lateral edges, *VA*, *VB*, etc, are 25 inches long. To the nearest tenth of an inch, calculate the height of pyramid *VABCDEFGH*.

2. The ratio of the volumes of two similar circular cylinders is 27: 64. What is the ratio of the diameters of their bases?

3. The surface areas of two cubes are in the ratio of 49: 81. If the volume of the smaller cube is 20, what is the volume of the larger cube?

4. Charlie built a treasure box. Lucy built a treasure box with dimensions twice as large as Charlie's. If it takes one-half gallon of paint to cover the surface of Charlie's box, how many gallons of paint would it take to paint Lucy's box? How many times more volume will Lucy's box hold than Charlie's?

5. Find the area of the regular polygon whose exterior angle is 45 and whose sides are 3.5 inches.

6. A triangle is defined by placing vectors [5, 7] and [-21, 15] tail to tail. Find its angles.

7. A 20-inch chord is drawn in a circle with a 12-inch radius. What is the *angular size* of the minor arc of the chord?

8. Find the lengths of both altitudes in the parallelogram determined by [2, 3] and [−5, 7].

9. To the nearest tenth of a degree, find the angle formed by placing the vectors [4, 3] and [-7, 1] tail-to-tail.

10. Suppose that square *PQRS* has 15-cm sides and that *G* and *H* are on *QR* and *PQ*, respectively, so that *PH* and *QG* are both 8 cm long. Let *T* be the point where *PG* meets *SH*. Find the size of angle *STG*, with justification.

11. (Continuation) Find the lengths of *PG* and *PT*.

12. Find the vertices and the area of the triangle formed by $y = |x - 3|$ and $-x + 2y = 5$.

13. Is it possible for a trapezoid to have sides of lengths 3, 7, 5, and 11?

14. It is given that the sides of an isosceles trapezoid have lengths 3 in, 15 in, 21 in, and 15 in. Make a diagram. Show that the diagonals intersect perpendicularly.

15. Triangle *ABC* has *AB* = *AC*. The bisector of angle *B* meets *AC* at *D*. Extend side *BC* to *E* so that *CE* = *CD*. Triangle *BDE* should look isosceles. Is it? Explain.

16. Find coordinates for the centroid of the triangle whose vertices are **(a)** (-1, 5), (-2, 8) and (3, 3); **b)** (2, 7), (8, 1) and (14, 11); **(c)** (a, p), (b, q) and (c, r).

1. Segments *AC* and *BD* intersect at *E*, so as to make *AE* twice *EC* and *BE* twice *ED*. Prove that segment *AB* is twice as long as segment *CD*, and parallel to it.

2. Suppose that *DRONE* is a regular pentagon and that *DRUM*, *ROCK*, *ONLY*, *NEAP*, and *EDIT* are squares attached to the outside of the pentagon. Show that decagon *ITAPLYCKUM* is equiangular. Is this decagon equilateral?

3. Let *RICK* be a parallelogram, with *M* the midpoint of *RI*. Draw the line through *R* that is parallel to *MC*; it meets the extension of *IC* at *P*. Prove that $CP = KR$.

4. Suppose that *PEANUT* is a regular hexagon, and that *PEGS*, *EACH*, *ANKL*, *NUMB*, *UTRY*, and *TPOD* are squares attached to the outside of the hexagon. Decide whether or not dodecagon *GSODRYMBKLCH* is regular and give your reasons.

5. Point P is marked inside regular pentagon *TRUDY* so that triangle *TRP* is equilateral. Decide whether or not quadrilateral *TRUP* is a parallelogram and give your reasons.

6. A triangle with sides 6, 8, and 10 and a circle with radius is r are drawn so that no part of the triangle lies outside the circle. How small can r be?

7. Diagonals *AC* and *BD* of regular pentagon *ABCDE* intersect at *H*. Decide whether or not *AHDE* is a rhombus, and give your reasons.

8. Let $A = (3, 1)$, $B = (9, 5)$, and $C = (4, 6)$. Your protractor should tell you that angle *CAB* is about 45 degrees. Explain why angle *CAB* is in fact exactly 45 degrees.

9. The sides of a polygon are cyclically extended to form *rays*, creating one exterior angle at each vertex. Viewed from a great distance, what theorem does this figure illustrate?

10. In trapezoid *ABCD*, *AB* is parallel to *CD*, and $AB = 10$, $BC = 9$, $CD = 22$, and $DA = 15$. Points *P* and *Q* are marked on *BC* so that $BP = PQ = QC = 3$, and points *R* and *S* are marked on *DA* so that $DR = RS = SA = 5$. Find the lengths *PS* and *QR*.

11. Triangle *ABC* has $AB = 12 = AC$ and angle *A* is 120 degrees. Let *F* and *D* be the midpoints of sides *AC* and *BC*, respectively, and *G* be the intersection of segments *AD* and *BF*. Find the lengths *FD*, *AD*, *AG*, *BG*, and *BF*.

12. The midpoints of the sides of a quadrilateral are joined to form a new quadrilateral. For the new quadrilateral to be a rectangle, what must be true of the original quadrilateral?

13. The vectors [8, 0] and [3, 4] form a parallelogram. Find the lengths of its altitudes.

14. Square *ABCD* has 8-inch sides, *M* is the midpoint of *BC*, and *N* is the intersection of *AM* and diagonal *BD*. Find the lengths of *NB*, *NM*, *NA*, and *ND*.

1. Parallelogram *PQRS* has *PQ* = 8 cm, *QR* = 9 cm, and diagonal *QS* = 10 cm. Mark *F* on *RS*, exactly 5 cm from *S*. Let *T* be the intersection of *PF* and *QS*. Find the lengths *TS* and *TQ*.

2. The parallel sides of a trapezoid are 12 inches and 18 inches long. The non-parallel sides meet when one is extended 9 inches and the other is extended 16 inches. How long are the non-parallel sides of this trapezoid?

3. Show that the area of a square is half the product of its diagonals. Then consider the possibility that there might be other quadrilaterals with the same property.

4. The dimensions of rectangle *ABCD* are *AB* = 12 and *BC* = 16. Point *P* is marked on side *BC* so that *BP* = 5, and the intersection of *AP* and *BD* is called *T*. Find the lengths of the four segments *TA*, *TP*, *TB*, and *TD*.

5. The altitude drawn to the hypotenuse of a right triangle divides the hypotenuse into two segments, whose lengths are 8 inches and 18 inches. How long is the altitude?

6. A triangle has two 13-cm sides and a 10-cm side. The largest circle that fits inside this triangle meets each side at a point of tangency. These points of tangency divide the sides of the triangle into segments of what lengths?

7. (Continuation) What is the radius of this circle?

8. In the middle of the nineteenth century, octagonal barns and sheds (and even some houses) became popular. How many cubic feet of grain would an octagonal barn hold if it were 12 feet tall and had a regular base with 10-foot edges?

9. Find the volume of a single sheet of paper.