Mathematics 225 Reference

AA similarity: Two *triangles* are sure to be similar if at least two of their angles are equal in size.

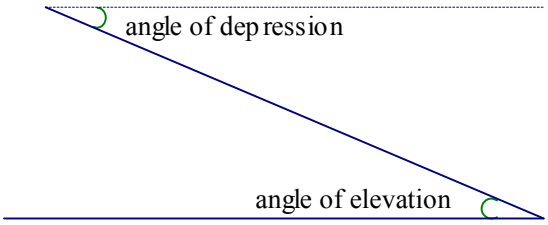
adjacent angles: Two angles with a common vertex that share a side but have no common interior points.

altitude: In a triangle, an altitude is a segment that joins one of the three vertices to a point on the opposite side, the intersection being perpendicular. In some triangles, it may be necessary to extend the side to meet the altitude. The *length* of this segment is also called an altitude, as is the distance that separates the parallel sides of a trapezoid.

angles can often be identified by a single letter, but sometimes three letters are necessary. The angle shown can be called *B*, *ABC*, or *CBA*.

angle of depression: Angle formed by a horizontal ray and a line-of-sight ray that is below the horizontal. See the diagram below.

angle of elevation: Angle formed by a horizontal ray and a line-of-sight ray that is above the horizontal. See the diagram above.



Angle-Angle-Side (corresponding): When the parts of one triangle can be matched with the parts of another triangle, so that two pairs of corresponding angles have the same sizes, and so that one pair of corresponding sides has the same length, then the triangles are congruent. This rule of evidence is abbreviated to AAS.

angle bisector: Given an angle, this ray divides the angle into two equal parts.

Angle Bisector Theorem: The bisector of any angle of a triangle cuts the opposite side into segments whose lengths are proportional to the sides that form the angle.

Angle-Side-Angle: When the parts of one triangle can be matched with the parts of another triangle, so that two pairs of corresponding angles have the same sizes, and so that the (corresponding) shared sides have the same length, then the triangles are congruent. This rule of evidence is abbreviated to ASA.

angular size of an arc: This is the size of the central angle formed by the radii that meet the endpoints of the arc.

arc: The portion of a circle that lies to one side of a chord is called an *arc*.

arc length: Given a circle, the length of any arc is proportional to the size of its central angle.

areas of similar figures: If two figures are similar, then the ratio of their areas equals the *square* of the ratio of similarity.

bisect: Divide into two pieces that are, in some sense, equal.

central angle: An angle formed by two radii of a circle.

centroid: The medians of a triangle are concurrent at this point, which is the balance point (also known as the *center of gravity*) of the triangle.

chord: A segment that joins two points on a circle is called a *chord* of the circle.

circle: This curve consists of all points that are at a constant distance from a *center*. The common distance is the *radius* of the circle. A segment joining the center to a point on the circle is also called a *radius*.

circumcenter: The perpendicular bisectors of the sides of a triangle are concurrent at this point, which is equidistant from the vertices of the triangle.

circumscribed circle: When possible, the circle that goes through all the vertices of a polygon.

collinear: Three (or more) points that all lie on a single line are *collinear*.

common chord: The segment that joins the points where two circles intersect.

complementary: Two angles that fit together to form a right angle are called complementary. Each angle is the *complement* of the other.

components describe how to move from one unspecified point to another. They are obtained by *subtracting* coordinates.

concentric: Two figures that have the same center are called *concentric*.

concurrent: Three (or more) lines that go through a common point are *concurrent*.

concyclic: Points that all lie on a single circle are called *concyclic*.

congruent: When the points of one figure can be matched with the points of another figure, so that corresponding parts have the same size, then the figures are called *congruent*, which means that they are considered to be equivalent.

converse: The converse of a statement of the form “if [something] then [something else]” is the statement “if [something else] then [something].”

convex: A polygon is called *convex* if every segment joining a pair of points within it lies entirely within the polygon.

coordinates: Numbers that describe the position of a point in relation to the origin of a coordinate system.

corresponding: Describes parts of figures (such as angles or segments) that have been matched by means of a transformation.

cosine ratio: Given a right triangle, the cosine of one of the acute angles is the ratio of the length of the side *adjacent* to the angle to the length of the hypotenuse. The word cosine is a combination of *complement* and *sine*, so named because the cosine of an angle is the same as the sine of the complementary angle.

CPCTC: *Corresponding Parts of Congruent Triangles are themselves Congruent*.

Crossed Chords Theorem: When two chords intersect inside a circle, the product of the lengths of the segments of one chord equals the product of the lengths of the segments of the other chord. Thus the value of this product depends only on the location of the point of intersection.

cyclic: A polygon, all of whose vertices lie on the same circle, is called *cyclic*. Also called an *inscribed polygon*.

decagon: A polygon that has ten sides.

diagonal: A segment that connects two nonadjacent vertices of a polygon.

diameter: A chord that goes through the center of its circle is called a *diameter*.

dilation: A similarity transformation, with the special property that all lines obtained by joining points to their images are concurrent at the same *central* point.

distance formula: The distance from (x_1, y_1) to (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_2 - y_1)^2}$. This formula is a consequence of the *Pythagorean Theorem*.

dodecagon: A polygon that has twelve sides.

dodecahedron: A polyhedron formed by attaching twelve polygons edge to edge. If the dodecagon is regular, each of its vertices belongs to three congruent *regular* pentagons.

equiangular: A polygon all of whose angles are the same size.

equidistant: A shortened form of *equally distant*.

equilateral: A polygon all of whose sides have the same length.

Euclidean geometry (also known as plane geometry) is characterized by its parallel postulate, which states that, *given a line, exactly one line can be drawn parallel to it through a point not on the given line*. A more familiar version of this assumption states that *the sum of the angles of a triangle is a straight angle*.

Euler line: The centroid, the circumcenter, and the orthocenter of any triangle are collinear.

exterior angle: An angle that is formed by a side of a polygon and the extension of an adjacent side. It is supplementary to the adjacent interior angle.

Exterior Angle Theorem: An exterior angle of a triangle is the sum of the two nonadjacent interior angles.

function: A function is a rule that describes how the value of one thing is determined uniquely by the value of another thing.

glide-reflection: An isometric transformation of a plane that leaves no single point fixed, but that does map a single line to itself. A glide-reflection thus maps points on either side of this line to the other side. Think of the footprints left by a person walking in a straight line.

Greek letters appear often in mathematics. Some of the common ones are α (alpha), β (beta), Δ or δ (delta), θ (theta), Λ and λ (lambda), μ (mu), π (pi), and Ω or ω (omega).

head: Vector terminology for the second vertex of a directed segment.

hexagon: a polygon that has six sides.

Hypotenuse-Leg: When the hypotenuses of two right triangles have the same length, and a leg of one triangle has the same length as a leg of the other, then the triangles are congruent. This rule of evidence is abbreviated to HL.

icosahedron: A polyhedron formed by attaching twenty polygons edge to edge. If the polyhedron is regular, each of its vertices belongs to five equilateral triangles.

icosidodecahedron: A polyhedron formed by attaching the edges of twenty equilateral triangles to the edges of twelve regular pentagons. Two triangles and two pentagons meet at each vertex.

image: The result of applying a transformation to a point P is called the *image of P*, often denoted P'. One occasionally refers to an *image segment* or an *image triangle*.

incenter: The angle bisectors of a triangle are concurrent at this point, which is equidistant from the sides of the triangle.

included angle: The angle formed by two designated segments.

inscribed angle: An angle formed when two chords meet at a point on the circle. An inscribed angle is *half* the angular size of the arc it intercepts. In particular, an angle that intercepts a semicircle is a *right* angle.

inscribed polygon: A polygon whose vertices all lie on the same circle; also called a *cyclic polygon*.

integer: Any whole number, whether it be positive, negative, or zero.

intercepted arc: The part of an arc that is found inside a given angle.

isometry: A geometric transformation that preserves distances. The best-known examples of isometries are *translations*, *rotations*, and *reflections*.

isosceles triangle: A triangle that has two sides of the same length. The word is derived from the Greek *iso* + *skelos* (equal + leg)

Isosceles Triangle Theorem: If a triangle has two sides of equal length, then the angles opposite those sides are also the same size.

isosceles trapezoid: A trapezoid whose nonparallel sides have the same length.

kite: A quadrilateral that has two pairs of congruent adjacent sides.

labeling convention: Given a polygon that has more than three vertices, place the letters around the figure in the order that they are listed.

lateral face: Any face of a pyramid or prism that is not a base.

lattice point: A point whose coordinates are both integers.

lattice rectangle: A rectangle whose vertices are all lattice points.

leg: The perpendicular sides of a right triangle are called its legs.

length of a vector: This is the length of any segment that represents the vector.

linear equation: Any straight line can be described by an equation in the form $ax + by = c$.

linear pair: Two adjacent angles whose sum is 180 degrees; Two angles that form a straight angle.

magnitude of a dilation: The nonnegative number obtained by dividing the length of any segment into the length of its dilated image. See *ratio of similarity*.

major/minor arc: Of the two arcs determined by a given chord, the smaller one is called *minor*, and the larger one is called *major*.

median of a triangle: A segment that joins a vertex of a triangle to the midpoint of the opposite side.

midline of a trapezoid: This segment joins the midpoints of the non-parallel sides. Its length is the average of the lengths of the parallel sides, to which it is also parallel. Also known as the *median* in some books.

Midsegment Theorem: A segment that joins the midpoints of two sides of a triangle is parallel to the third side, and is half as long.

midpoint: The point on a segment that is equidistant from the endpoints of the segment. If the endpoints are (*a, b*) and (*c, d*), the midpoint is $\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$

mirror: See *reflection*.

negative reciprocal: One number is the negative reciprocal of another if the product of the two numbers is -1.

octagon: a polygon that has eight sides.

opposite: Two numbers or vectors are opposite if they differ in sign. For example, 17.5 is the opposite of -17.5, and $[2, -11]$ is the opposite of $[-2, 11]$.

orthocenter: The altitudes of a triangle are concurrent at this point.

parallel: Coplanar lines that do not intersect. When drawn in a coordinate plane, they are found to have the same slope, or else no slope at all. The shorthand // is often used.

parallelogram: A quadrilateral that has two pairs of parallel sides.

pentagon: a polygon that has five sides.

perpendicular: Coplanar lines that intersect to form a right angle.

perpendicular bisector: Given a line segment, this is the line that is perpendicular to the segment and that goes through its *midpoint*. The points on this line are all *equidistant* from the endpoints of the segment.

point-slope form: A non-vertical straight line can be described by $y - y_0 = m(x - x_0)$ or by $y = m(x - x_0) + y_0$. One of the points on the line is (x_0, y_0) and the slope is m.

postulate: A statement that is accepted as true, without proof.
prism: A three-dimensional figure that has two congruent and parallel *bases*, and parallelograms for its remaining *lateral faces*. If the lateral faces are all rectangles, the prism is a *right prism*. If the base is a regular polygon, the prism is also called *regular*.

probability: A number between 0 and 1, often expressed as a percent, that expresses the likelihood that a given event will occur. For example, the probability that two coins will *both* fall showing heads is 25%.

proportion: An equation that expresses the equality of two *ratios*.

pyramid: A three-dimensional figure that is obtained by joining all the points of a polygonal *base* to a *vertex*. Thus all the lateral faces of a pyramid are triangles. If the base polygon is regular, and the lateral edges are all congruent, then the pyramid is called *regular*.

Pythagorean Theorem: The square on the hypotenuse of a right triangle equals the sum of the squares on the legs. If a and b are the lengths of the legs of a right triangle, and if c is the length of the hypotenuse, then these lengths fit the Pythagorean equation $a^2 + b^2 = c^2$.

quadrant: one of the four regions formed by the coordinate axes. Quadrant I is where both coordinates are positive, and the other quadrants are numbered (using Roman numerals) in a counterclockwise fashion.

quadratic formula: $x = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $x = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$ are the two solutions to $ax^2 + bx + c = 0$.

quadrilateral: a four-sided polygon.

ratio of similarity: The ratio of the lengths of any two corresponding segments of similar figures.

ray A ray is a line bounded at one end and infinite at the other.

reflection: A transformation of a plane that has a line of fixed points. A reflection maps points on either side of this line (the *mirror*) to the other side.

regular: A polygon that is both equilateral and equiangular.

rhombus: An equilateral quadrilateral.

right angle: An angle that is its own supplement.

rotation: An isometric transformation of a plane that leaves a single point fixed.

SAS similarity: Two triangles are certain to be similar if two sides of one triangle are proportional to two sides of the other, and if the included angles are equal in size.

scalar: In the context of vectors, this is just another name for a number.

scalene: A triangle no two of whose sides are the same length.

segment: That part of a line that lies between two designated points.

Sentry Theorem: The sum of the exterior angles (one per vertex) of any polygon is 360 degrees.

Shared Altitude Theorem: If two triangles share an altitude, then the ratio of their areas is proportional to the ratio of the corresponding bases.

Shared Base Theorem: If two triangles share a base, then the ratio of their areas is proportional to the ratio of the corresponding altitudes.

Side-Angle-Side: When the parts of one triangle can be matched with the parts of another triangle, so that two pairs of corresponding sides have the same lengths, and so that the (corresponding) angles they form are also the same size, then the triangles are congruent. This rule of evidence is abbreviated to just SAS.

Side-Side-Angle: Insufficient grounds for congruence. See *Hypotenuse-Leg*, however.

Side-Side-Side: When the parts of one triangle can be matched with the parts of another triangle, so that all three pairs of corresponding sides have the same lengths, then the triangles are congruent. This rule of evidence is abbreviated to just SSS.

similar: Two figures are similar if their points can be matched in such a way that all ratios of corresponding lengths are proportional to a fixed *ratio of similarity*. Corresponding angles of similar figures must be equal in size.

sine ratio: Given a right triangle, the sine of one of the acute angles is the ratio of the length of the side *opposite* the angle to the length of the hypotenuse.

skew lines: Non-coplanar lines that do not intersect.

slope: The slope of the segment that joins the points (x_1, y_1) and (x_2, y_2) is $\frac{y_2 - y_1}{x_2 - x_1}$.

slope-intercept form: Any non-vertical straight line can be described by an equation that takes the form $y = mx + b$. The slope of the line is m, and the y-intercept is b.

SSS similarity: Two triangles are *similar* if their sides are proportional.

supplementary: Two angles that fit together to form a straight line are called *supplementary*. Each angle is the *supplement* of the other.

tail: Vector terminology for the first vertex of a directed segment.

tail-to-tail: Vector terminology for directed segments with a common first vertex.

tangent ratio: Given a right triangle, the tangent of one of the acute angles is the ratio of the side opposite the angle to the side adjacent to the angle.

tangent and slope: When an angle is formed by the positive x-axis and a ray through the origin, the *tangent* of the angle is the *slope* of the ray. Angles are measured in a counterclockwise sense, so that rays in the second and fourth quadrants determine negative tangent values.

tangent to a circle: A line that touches a circle without crossing it. Such a line is perpendicular to the radius drawn to the point of tangency.

tesselate: To fit non-overlapping tiles together to cover a planar region.

tetrahedron: A pyramid whose four faces are all triangles.

Three Parallels Theorem: Given three parallel lines, the segments they intercept on one transversal are proportional to the segments they intercept on any transversal.

transformation: A *function* that maps points to points.

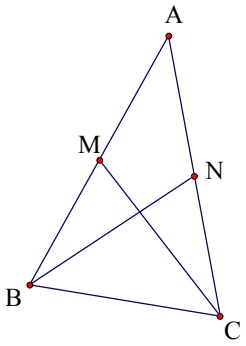
translate: To slide a figure by applying a vector to each of its points.

transversal: A line that intersects two other lines in a diagram.

trapezoid: A quadrilateral with exactly one pair of parallel sides. If the non-parallel sides have the same length, the trapezoid is called *isosceles*.

triangle inequality: The sum of the lengths of two sides of a triangle is greater than the length of the third side.

two-column proof: A way of outlining a geometric deduction. Steps are in the left column, and supporting reasons are in the right column. For example, here is how one might show that an isosceles triangle ABC has two medians of the same length. It is given that $AB = AC$ and that M and N are the midpoints of sides AB and AC, respectively. The desired conclusion is that medians CM and BN have the same length.



$AB = AC$	given
$AM = AN$	M and N are midpoints
$\angle MAC = \angle NAB$	shared angle
$\Delta MAC \cong \Delta NAB$	SAS
$CM = BN$	CPCTC

Two Tangent Theorem: From a point outside a circle, there are two segments that can be drawn tangent to the circle. These segments have the same length.

unit circle: This circle consists of all points that are 1 unit from the origin O of the xy-plane. Given a point P on this circle, the coordinates of P are the *cosine* and the *sine* of the counterclockwise angle formed by segment OP and the positive x-axis.

unit square: Its vertices are (0, 0), (1, 0), (0, 1), and (1, 1).

Varignon parallelogram: Given any quadrilateral, this is the figure formed by connecting the midpoints of consecutive sides.

vectors have *magnitude* (size) and *direction*. Visualize them as directed segments (arrows). Vectors are described by *components*, just as points are described by coordinates. The vector from point A to point B is often denoted $\overrightarrow{AB}$ or abbreviated by a boldface letter such as **u**, and its magnitude is often denoted $\left|\overrightarrow{AB}\right|$ or $|\mathbf{u}|$.

velocity: A vector obtained by dividing a displacement vector by the elapsed time.

vertex: A labeled point in a figure. The plural is *vertices*, but "vertice" is not a word. The point on a parabola that is closest to the focus is also called the vertex.

vertical angles: Two non-adjacent angles that share a vertex and are formed by the intersection of two lines.

volume of a prism: This is the product of the *base area* and the *height*, which is the distance between the parallel base planes.

volume of a pyramid: This is one third of the product of the *base area* and the *height*, which is the distance from the vertex to the base plane.

volumes of similar figures: If two three-dimensional figures are similar, then the ratio of their volumes equals the *cube* of the ratio of similarity.