

Motivational Problems for Parametric Equations and Polar and Rectangular Vectors

1. I have been observing the motion of a bug on my graph paper. When I started watching, it was at the point $(1, 2)$. Ten seconds later it was at $(3, 5)$. Another ten seconds later it was at $(5, 8)$. After another ten seconds it was at $(7, 11)$.
 - (a) Draw a picture that illustrates what is happening.
 - (b) Write a description of any pattern that you notice. What assumptions are you making?
 - (c) Where was the bug 24 seconds after I started watching it?

2. From its initial position at $(3,4)$, an object moves linearly, reaching $(9,8)$ after two seconds and $(15,12)$ after four seconds.
 - (a) Predict the position of the object after six seconds; after nine seconds; after t seconds.
 - (b) Is there a time when the object is equidistant from the coordinate axes? If so, where is it?

3. The x - and y -coordinates of a point are given by the equations shown below.
$$x = 2 + 2t$$
$$y = 5 - t$$

The position of the point depends on the value assigned to t . Use your graph paper to plot points corresponding to the values $t = -4, -3, -2, -1, 0, 1, 2, 3$, and 4. Do you recognize any patterns? Describe them.

4. (Continuation) Plot the following points on the coordinate plane: $(1,2)$, $(2,5)$, $(3,8)$. Write equations, similar to those in the preceding exercise, that produce these points when t -values are assigned. There is more than one correct answer.
5. In a dream, Blair is confined to a coordinate plane, moving along a straight line with a constant speed. Blair's position at 4 a.m. is $(2,5)$ and at 6 a.m. it is $(6,3)$. What is Blair's position at 8:15 a.m. when the alarm goes off?
6. At noon one day, Corey decided to follow a straight course in a motor boat. After one hour of making no turns and traveling at a steady rate, the boat was 6 miles east and 8 miles north of its point of departure. What was Corey's position at two o'clock? How far had Corey traveled? What was Corey's speed?
7. (Continuation) Assuming that the gas tank initially held 12 gallons of fuel, and that the boat gets 4 miles to the gallon, how far did Corey get before running out of fuel? When did this happen? How did Corey describe the boat's position to the Coast Guard when radioing for help?
8. The Krakow airport is 5 km north and 3 km west of the city. At 1 p.m., Zuza took off in a Cessna 730. Every six minutes, the plane's position changed by 7 km north and 9 km east. At 2:30 p.m., Zuza was flying over the town of Jozefow. Where is this town, in relation to the center of Krakow (assumed at an origin)? Describe Zuza's position after t hours of flying.
9. When the translation by vector $[2, 5]$ is followed by the translation by vector $[5, 7]$, the net result can be achieved by applying a *single* translation; what is its vector?

10. The x-component of a velocity vector is 15 mph, and the speed is 17 mph. Find all possible values for the y-component.
11. Consider the vector $[3,4]$ with tail end at the origin and front end at the point $(3,4)$. How might you find the measure of the angle (in degrees) that this vector makes with the x-axis? What is that angle?
12. The *sum* of two vectors $[a, b]$ and $[p, q]$ is defined as $[a+p, b+q]$. Find components of the following vectors (a) $[2, 3] + [-7, 5]$ (b) $\frac{1}{2} [2, 6] + [-3, 7]$
13. (continuation from 11) Rewrite the vector $[3,4]$ in terms of the angle you found in #11. In other words, what is the x component of this vector in terms of the angle, and what is the y component in terms of the angle?
14. Simplify the sum of vectors $\vec{AB} + \vec{BC} + \vec{CD}$. Draw yourself a diagram for some help, but do not assign numerical values to the vectors.
15. Rewrite the polar vector $[5\cos 30, 5 \sin 30]$ in rectangular form. What is the magnitude of this vector? Explain your findings. In general, when given the polar vector $[r \cos \theta, r \sin \theta]$, find the magnitude of the vector.
16. In quadrilateral $ABCD$, it is given that $\vec{AB} = \vec{DC}$. What kind of a quadrilateral is $ABCD$? What can be said about the vectors AD and BC ?
17. Mark a point on your graph paper. Define vector \mathbf{u} by moving 5 units to the right and 2 units up. Define vector \mathbf{v} by moving 1 unit to the right and 3 units down. Diagram the vectors $\mathbf{u}+\mathbf{v}$, $\mathbf{u}-\mathbf{v}$, $\mathbf{u}+2\mathbf{v}$, and $2\mathbf{u}-3\mathbf{v}$.
18. Draw a parallelogram. Choose one of its vertices and let \mathbf{u} and \mathbf{v} be the vectors defined by the sides that originate at that vertex. Draw $\mathbf{u}+\mathbf{v}$ and $\mathbf{u}-\mathbf{v}$. The vectors \mathbf{u} and \mathbf{v} represent the sides of the parallelogram; what do $\mathbf{u}+\mathbf{v}$ and $\mathbf{u}-\mathbf{v}$ represent?
19. Draw an acute triangle ABC , and let M and N be the midpoints of sides AB and AC , respectively. Let $\mathbf{u} = \vec{AB}$ and $\mathbf{v} = \vec{AC}$. Express BC and MN in terms of \mathbf{u} and \mathbf{v} .

Extra:

20. An airplane that took off from its airport at noon ($t = 0$ hrs) moves according to the formula $(x, y, z) = (15, -20, 0) + t [450, -600, 20]$. Twelve minutes later, it flew over Bethlehem. Where is the airport in relation to Bethlehem, and how high (km) was the airplane above the town?
21. Chris and Kim are cruising space, according to the equations $C(t) = (27, 68, 70) + t[4, -7, 4]$, and $K(t) = (23, 11, 34) + t[4, 1, 8]$. Show that their paths intersect, but that there is no collision. Who reaches the intersection first? Who is moving faster?